STUDENT COMPREHENSION ABOUT LINE AND ROWFROM APOS THEORY POINT OF VIEW

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Abstract:

The way of individual student in constructing mathematical concept he/she learned can be through different ways. The development of student comprehension in constructing that concept can be analyzed by APOS (Action, Processes, Object, and Scheme) Theory. This theory can be used to describe individual's scheme development for certain mathematic topic. This paper will discuss how to reveal student ability in constructing or reconstructing action, process, and object of mathematic and organizing it in scheme which is used to solve line and row problem.

Keywords: comprehension, action, process, object and scheme

1. INTRODUCTION

In mathematic, the basic object learned is abstract, also often called mental object. Because the abstractness of the object, this become one of factor which cause the difficulty to learn mathematic. The underlying aspect in learning mathematic is to implant mathematical concept based on comprehension. Someone who learn mathematic should achieve depth comprehension in order to apply it in real situation and experience the benefit of mathematic in daily life.

According Dubinsky (2000), comprehension toward a mathematical concept is the result of construction and reconstruction toward mathematical objects. Construction and reconstruction is done through activities of actions, processes, and objects of mathematic which is organized in a schema to solve mathematical problem. This is can be analyzed through analyses of genetic decomposition as operation from APOS (Action, Process, Object and Scheme) theory. APOS theory is a constructivist theory about how the achievement of concept learning or mathematical principle occurred, which can be used as elaboration about mental construction from action, process, object and scheme. This theory can be used as analyses tool to describe one's scheme development toward a mathematical topic which is totality from knowledge related with that object. The scheme development is a process which is dynamic and always change.

Line and row is one of mathematical topic which is learned in class 1 of Senior High School. Line and row used much in daily life. Line and row concepts also can be used in learning advance mathematic material, for instance algebra function limit. Line concept also can be used to help in finding the pattern, forming the hypothesis, developing critical thinking ability, and prove a mathematical conjecture. Therefore, line and row is one of essential topic, so the right comprehension is needed.

From several researchers which have done this so far, it can be known that there are still many students who experience the difficulty in comprehending line and row. Therefore how far and what is student doing when he/she construct mathematical concept or solve mathematical problem which is related with line and row is needed to be studied.

In this paper, the author want to discuss the way of tracking student comprehension for line and row topics. Sub topic discussed is arithmetic line and row so what means by line and row in this paper is arithmetic line and row.

2. DISCUSION

A. Comprehension

Comprehension can be defined as depth understanding. According Miozek (in Nurdin, 2005), comprehension is a process to comprehend certain meaning and the ability to use it in another situation. Dubinsky (2000) state that comprehension about mathematical concept is the result of construction and reconstruction from mathematical objects which is done through activities of action, process, and object which is coordinated in a scheme.

According Piaget (Suparno, 2001), process of forming scheme involved two activities, namely assimilation and accommodation. Assimilation is process to absorb new experience (knowledge) into existing scheme. Whereas accommodation is process to absorb new experience (knowledge) by modifying existing scheme or even forming experience (knowledge) which is really new. Comprehension in this article defined as student's ability to construct or reconstruct mathematical action, process, and object and organize it in scheme which is used to solve line and row problem.

B. Mental Constructions in Comprehending Mathematical Concept

Dubinsky et all (2000) develop APOS theory which is elaboration result of reflective abstraction which is introduced by Piaget in explaining the development of logical thinking in children. Dubinsky expand this idea to explain the development of higher order mathematical thinking in students college. Basically, APOS theory can be used to explain the development of mathematical thinking of each individual, so the author of this paper choose to study and apply APOS theory to explain the development of Senior High School students' mathematical thinking.

APOS theory assumed that mathematical knowledge which is possessed by someone is the result of interaction outcome with other people and the result of his/her mental constructions in comprehending mathematical ideas. Those mental constructions

are: action, process, object and scheme which is abbreviated as APOS. Frequently, some constructions is reconstruction from existing one, but its reconstruction do not exactly the same with the reconstruction which had already existed before. The terms construction and reconstruction which is intended here resemble with Piaget's terms namely accommodation and assimilation (Asiala, et all, 2004). APOS theory is very good to be used to comprehend students college learning in various mathematic topics in higher education, such as calculus, abstract algebra, statistic, discrete mathematic, etc (Dubinsky, 2000).

According Dubinsky (2000), comprehending the mathematical concept is began by manipulating existing mental construction or manipulating physical object to form action. Action then is interiorized (contemplated) to form the process which then is encapsulated (crystallized) to form the object. The object can be decomposed again to become process. Finally, action, process, and object can be organized in scheme. The following is brief description for each APOS mental construction.

Action is transformation which is experienced to occur in individual thinking as result of stimuli from outside. The stimuli for instance in the form of implementing instructional stages for an operation. This action is activity in the form of physical repetition or mental manipulation which basing algorithm explicitly. Action can be intended as physical or mental transformation from object to obtain another object.

When an action is done repeatedly, and individual reflect on it, so the action is interiorized to become process, that is an internal construction which is made by doing the action, but now not directed by stimulus from outside. Individual who has construct "process of concept" can decompose or even reverse steps from transformation without really do it.

The object is constructed from process when individual reflect on operation which is applied on process for certain concept, become aware toward process as totality and really can construct that transformation, so that individual encapsulate process to become object.

The collection of action, process, object and another scheme which is connected integrally and organized structurally in individual thinking is called scheme.

Construction which relate an action, process, object which is separate that result in certain scheme is called thematization. This scheme can be relied in dealing with problem in mathematic. The difference between scheme with another mental constructions resemble the difference in biology between organ and cell. Both is object, but organ (scheme) give necessities in order that cell to be functioned as should be. Scheme of individual is whole knowledge which he connected consciously or unconsciously with certain mathematical concept. Individual can posses the scheme for function, the scheme for derivative, etc. The scheme itself can be treated as object and contain in scheme organization in higher level.

C. Scheme Development

APOS theory can be used directly to analyze a researcher data (Dubinsky dan Fauvel, J. 2000). Researchers can compare subject's success and failure for mathematical task through certain mental construction which they can or cannot do.

In searching and analyzing how student learn mathematical concepts, mental construction elements of action, process and object is very essential element to be noticed by researcher. Description that is result from concept analysis in that construct is called genetic decomposition from concept (Dubinsky, 2000). Whereas Asiala, et al (2004) stated that analysis of genetic decomposition is analyses toward structured collection from mental activities of action, process and object which is done by someone to describe how mathematical concept and principle can be developed in one's thinking. Therefore, analyses of genetic decomposition is analyses toward genetic decomposition in responding a mathematical problem based on APOS theory framework.

Dubinsky (2000) and Barker, et al (2000) developed APOS theory as adaptation to some Piaget opinion about one's knowledge development. Piaget and Garcia (in Barker, et al, 2000) stated that knowledge grow and develop based on certain mechanism which comprise three levels (stages) called *triad.* Based on above explanation, genetic decomposition analyses in this paper is defined as analyses toward student comprehension in responding line and row problem by based on APOS theory. Furthermore, triad stages from Piaget and Garcia is used in analyzing student's comprehension level about mathematical concept.

Triad occur in fixed order which is hierarchies and functional. That order is *intra* stage, *inter* stage, and *trans* stage. In intra stage, student concentrate attention on action or operation which can be repeated, but less able to connect action with condition system which make him/her able to broaden its application. The explanations in this stage still local and special. In this stage, student recognize the object not as important thing, and its form resemble the simple generalization form.

In inter stage, student realize about the relation which is occurred in object and be able to conclude based on initial process with some comprehension, and another operation as a result, or only able to coordinate with the same operations. This process make students are able to group a system by using method which incorporate new transformation.

In trans stage, student is able to arrange an awareness of scheme completeness and able to achieve new global properties which cannot be accessed in another stage. Student in this stage has ability to construct whole structure which is found (action, process, object, and another scheme) interrelated and form a coherent scheme.

One's scheme development toward a mathematical topic can be described by using APOS theory. Then, scheme development is analyzed to find out triad stages of one's scheme development. According Dubinsky (in Zazkis and Campbell, 1966), student's scheme development in intra stage is indicated by ability to interiorizing an action toward process. Scheme development in inter stage is indicated by ability to encapsulate a process to object. Whereas student's scheme development in trans stage is indicated by ability to thematizing object to scheme.

D. Application of APOS Theory Framework to Line and Row

In this section, brief description about application of APOS theory framework will be given by analyses of genetic decomposition which is integrated with triad stages from Piaget and Garcia to line and row concept. Table 1 and 2 is presented below which contain criteria of student's comprehension about line and row based in APOS theory framework (adopted from Nurdin, 2005 : 17 – 18).

Table 1. Criteria of Student Comprehension about Arithmetic Line

Topic	APOS Theory Framework	Criteria
Arithmetic Line	Action	Student capable to - Distinguish a line from another line by noticing pattern from some suku toward a line. - Assert the difference between a line with another line.
	Process	Student capable to explain the way to determine certain suku from a line.
	Object	Student capable to - Assert the definition of arithmetic line. - Assert example of an arithmetic line. - Assert the relation between one suku with another suku arithmetic line.
	Scheme	Student capable to: - Connect action, process, object of another arithmetic line that is linear function graphic. - Determine certain suku from arithmetic line by connecting action, process, object of arithmetic line.

Table 2. Criteria of Student Comprehension about Arithmetic Row

Topic	APOS Theory Framework	Criteria
Arithmetic Row	Action	Student capable to: - Distinguish a line with row by noticing number structure. - Assert the difference between line and row.
	Process	Student capable to explain

	Object Scheme	the way to determine the number of first suku from a line. Student capable to - Assert definition of arithmetic row. - Assert example of arithmetic row. - Assert example of row which is not arithmetic row. Student capable to - Connect action, process, object of arithmetic row with another mathematical object, namely arithmetic line concept and sigma notation. - Determine the number of certain suku .from a line by connecting action, process, object of arithmetic line and linear equation system.

APOS theory framework for line and row whose criteria has been determined in two tables above is as follow.

Action:

Action is physical and mental manipulation which can be repeated in transforming object by one way. For example, students is given the problem "How is the sum of 7 first sukus from line 5, 9, 13, 17, 21, ….?" Student action toward that problem which can be done is as follow. Summing numbers in that line until the 7^{th} suku, that is summing 5+9+13+17+21+25+29 = 119. So, student do the activity actively to find the sum of certain suku from a line by way of summing numbers in certain line.

Interiorization: from action to process

Interiorization is a change from a procedural activity to be able to do this activity again in imagining some understanding which is influential toward condition which is resulted. That change is used to distinguish an action from process, that is activity determine a line suku is interiorized as a process where that action will be done, but not really done. For instance, "How is the sum of 7 first suku from line of 5, 9, 13, 17, 21….?" In interiorizing the seeking of sum of that 7 first suku, student do not do the action, but do it in imagination and capable to explain the process of determining the sum of 7 first suku from that line, even though she/he still use the way of summing numbers which are existed in that line. Therefore, student can imagine and explain that the sum of 7 first suku from line of 5, 9, 13, 17, 21,….is obtained by adding numbers in line, that is 5+9_13_17_21_25_29 = 119.

If student has able to tell and explain about the way to obtain answer from determining the sum of a suku by summing numbers which are existed in line, so the level of student comprehension is in the intra stage.

Encapsulation: from process to object

If a process can be transformed by an action, so it is said that process has been encapsulated to become object. Encapsulation of process to determine a suku from line is indicated when student capable to show that this line has certain properties and characteristic, a line suku has linked with the next suku in certain category. Based on characteristic of known line, student is able to determine whether that line is included into certain line category. For example, the question is asked "How is the sum of 7 first suku from line of 5, 9, 13, 17, 21,…?" Student who has been encapsulated line as object is able to explain that this line is arithmetic line, because has characteristic of difference between two sukus in order is constant, which is called difference (d), that is b = 4, and first suku (u_1) = 5, therefore the sum of 7 first suku can be determined by using formula which is obtained from definition of arithmetic row, that is $S_7 = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7$ or $S_7 = 5+9+13+17+21+25+29 = 119$.

If student capable to determine the sum of certain suku from line by noticing the characteristic of line and relate it by concept of certain line and row, and able to give example and non example from row and line, so the level of student comprehension is in the inter stage.

Thematization: from object to scheme

Thematization is construction which linked action, process, and object which is separated for certain object in order to generate a scheme. Thematization of first suku n sum from line as a scheme involve special relation between row of line with sigma notation. A student is said has been able to thematize the row of line as scheme, if she/he can show the sum of first suku n from a line by linking it with sigma notation. For example, she is given the problem "How is the sum of 7 first suku from line of 5, 6, 13, 17, 21, ….?" Student who thematize arithmetic line is able to explain that the sum of 7 first suku from that line is a process of seeking the sum of 7 first suku from arithmetic line, because that line pattern has characteristic of arithmetic line, and capable to link the sum of 7 first suku with concept of sigma notation, that is $S_7 = \sum_{i=1}^{7} u_i = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7$ atau $S_7 = 5+9+13+17+21+25+29 = 119$.

Student who capable to link the relation between concept of arithmetic line row with another object (in this case is sigma notation) is said has comprehension in trans level.

3. CONCLUSION

Based on APOS theory, student comprehension toward line and row problem can be tracked. By genetic decomposition analysis, the development of student comprehension can be revealed. Scheme development of each student for line and row topic can be different so its scheme development can be mapped into one of triad stage. The study result in revealing the development of student comprehension can be followed up in learning-teaching process activity by chosing appropriate method for learning in class by

noticing where the point of student difficulty/inability in comprehending line and row problem.

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