Relationship between Scientific Reasoning Skills and Mathematics Achievement among Malaysian Students

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Abstract

The role of reasoning in mathematical performance is a continuing topic of interest for researchers in mathematics education. This present study explored the link between scientific reasoning skills and mathematics performance as measured by students' responses to a series of novel problems. Results indicated the existence of a moderate positive correlation between the two variables. All participating students exhibited low level of scientific reasoning. Despite this, students in the high-achievement group performed significantly better than their peers in the low-achievement group in the mathematics test. The results suggest that while scientific reasoning is necessary, these set of skills may not fully explain reasonings that underpin mathematical problem solving among Malaysia secondary students. We draw implications for instructions to support the development and use of reasoning in mathematical learning in Malaysian classrooms.

Keywords: reasoning, scientific reasoning skills, mathematics problem solving, Malaysian classrooms, secondary mathematics, knowledge transfer, gender

1. Introduction

There is a consensus about the need to support higher level engagement of secondary mathematics, specifically in mathematical problem solving (Santos-Trigo & Moreno-Armella, 2013). However, the nature of engagement with mathematics is a complex processes. One way to examine student engagement is by analyzing the processes that students activate during mathematical performance. In this study, we focus on reasoning and the transfer of reasoning among a cohort of high school students during problem solving, an area of higher order thinking skills that has not received a great of attention (Bieda, Ji, Drwencke & Picard, 2013; Stylianides, Stylianides & Shiling-Traina, 2013).

In discussions about cognitive processes that support mathematical learning and problem solving, a category of skills, namely, higher order thinking skills, is now emerging to be a significant area of interest for researchers and teachers. The development of higher order thinking skills is necessary in order to facilitate the transfer of students' prior mathematical knowledge. With calls for increasing attention to creativity, proof development and logical arguments, the study of higher order thinking skills could improve current understandings about processes that are involved during the search for solution (Schoenfeld, 1992, 2013; Bieda *et al*., 2013; Lester, 2013; Tularam, 2013).

1.1 Higher Order Thinking Skills: Malaysian Context

The Malaysian Government has given the development of scientific talent a national priority. Towards that end, the Malaysian Mathematics Integrated Curriculum for Secondary School focuses on fostering critical thinking and creativity, reasoning and higher order thinking skills. Mathematics at a higher levels of the school curriculum involves the use of abstract concepts which, in turn, requires formal logical reasoning that draws on concepts and principles.

Findings of international studies of student achievement such as Trends in Mathematics and Science Study [TIMSS] and Programme of International Student Assessment [PISA] showed that most high school students in Malaysia had not reached a satisfactory level of higher order thinking. In the analyses of Malaysian students' performance in TIMSS in a number of years, Mullis, Martin, Foy & Arora (2012) found that only 2-10% of the students are capable of interpreting the information, drawing conclusions and generalization in solving complex problems – activities that collectively reflect low levels of activation of higher order thinking. Mullis and colleagues also showed that 60% of students achieved the low international benchmark. These results suggest that the students understand the basic mathematical concepts but, in general, they are not able to transfer that knowledge to non-routine problem situations (Ministry of Education [MOE], 2013). Likewise, another international study, namely, the PISA 2009 showed that Malaysia students' performances were located in the bottom one third of all the 74 participating countries (Walker, 2011).

As in the TIMSS study, PISA's report for mathematics achievement showed that only a small proportion (8%) of Malaysian students achieved advanced levels of thinking. Overall, trends in TIMSS and PISA provide evidence of Malaysian students' continuing difficulty in solving mathematical tasks which involve complex interpretation and synthesis – key aspects of higher order thinking. The solution of complex mathematical problems involves the transfer of prior learning to new contexts, and this transfer, we argue, can be facilitated by the acquisition of higher order thinking skills.

In addition, the gender gap in mathematics achievement among Malaysian students had widened over the last five years (MOE, 2013). Girls have consistently outperformed boys at every level. The gap in performance is evidenced at the Primary School Assessment Test (PSAT) level and increased over a student's lifetime up to university level, where females comprise approximately 70% of the cohort. While this phenomenon is not unique to Malaysia, it does require attention to ensure that the country does not have a cohort of 'lost boys' who either leave school early or with low attainment levels. The trend in the performance of girls vs boys is also evident in international measures of mathematics attainment. For example, PISA 2012 international studies have shown that Malaysia is one out of five countries where girls outperformed boys in mathematics (Organisation for Economic Co-operation and Development [OECD], 2013). Similarly, the TIMSS 2011 results indicated that Malaysian Year 8 girls outperformed boys in mathematics in tasks that involved application and reasoning (Mullis *et al*., 2012). The findings of MOE, TIMSS and PISA are significant in that they generated important directions for future study in better understanding gender-related differences in mathematical performance. We take up this issue by analysing the link between reasoning and performance in the solution of mathematical problems by boys and girls.

The Malaysian situation highlighted two core issues. Firstly, there is limited evidence of use of higher order thinking skills for Malaysian students when they attempt to perform mathematical tasks. Secondly, the difference between girls and boys in mathematical performance has not been analysed in terms of processes that underpin their performances. The present study is grounded in the above issues by exploring the link between the higher order thinking skills and mathematics achievement. Specifically, we focused on a range of reasoning skills called Scientific Reasoning Skills [SRS] and examined Gender as a variable in analyzing mathematical performance and reasoning skills.

2. Literature Review

This section is organised in five parts: reasoning, reasoning in mathematics, reasoning as specific and general knowledge in mathematical problem solving, SRS, and SRS and knowledge transfer in mathematical performance. Each part is interrelated and focused on a major issue- the link between reasoning skills and mathematical performance.

2.1 Reasoning

While reasoning has been defined in different ways, there is a common strand in the conceptualisation of this process. Sternberg (1977, p. 99) characterised reasoning in the following manner:

We reason analogically whenever we make a decision about something new in our experience by drawing a parallel to something old in our experience. When we buy a new pet hamster because we liked our old one or when we listen to a friend's advice because it was correct once before, we are reasoning analogically.

In other words, when we have to make a decision referring to content that is unknown or in a surrounding that is new to us, we tend to relate it to similar past experiences in finding an answer. Accordingly without reasoning, previously acquired knowledge and experiences could not be applied or applied effectively to new situations.

Leighton (2004) defined reasoning as the process of drawing conclusions in order to achieve goals, thus informing problem-solving and decision-making behaviour. She explained that reasoning acts as a mediator thus may not often be acclaimed as it works behind the scene and it shadows more observable functions in problem solving and making-decision.

Thus understanding reasoning processes and establishing their relationship to mathematical performance are important tasks.

2.2 Reasoning in Mathematics

Current research emphasises the importance of students engaging in reasoning in all mathematical disciplines (National Council of Teachers of Mathematics [NCTM], 2009, 2010; Bieda *et al*., 2013; Santos-Trigo & Moreno-Armella, 2013; Stylianides *et al*., 2013). Ball and Bass (2003, p. 42) argued that “mathematical reasoning is inseparable from knowing mathematics with understanding.” Several scholars have elaborated on the connection between learning mathematics with understanding and reasoning. Lakatos (1976) noted that complete mathematical understanding includes the engaging processes of thinking, in essence doing what makers and users of mathematics do: framing and solving problems, patterns recognition, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on. This observation about understanding was recently reaffirmed by Schoenfeld (2013) in which he commented that one variable seemed to have strongest impact on students learning was the amount of time students spent in explaining their ideas, which clearly resonate students’ engagement with reasoning skills.

Reasoning in mathematics has undergone a variety of definitions and terms. Reasoning-and-proving describes a family of investigation activities of whether and why “things work” in various mathematical domains such as algebra, geometry, etc. (Stylianides, 2008). Reasoning as “the process of drawing conclusions on the basis of evidence or stated assumptions” (NCTM, 2009, p. 4). Reasoning involves engaging in processes to generalise mathematical phenomena and/or conjecturing about mathematical relationships (Bieda *et al*., 2013). Some researchers often refer reasoning as decomposes of two distinct processes: deductive and inductive reasoning skills (Barkl, Porter & Ginns, 2012). Over the years numerous models of reasoning have been put forward to fill the gap between inductive and deductive argumentation (e.g. representational reasoning (Simon, 1996), abductive reasoning (Cifarelli, 1999) and plausible reasoning (Polya, 1954). Such definitions and terms of reasoning differ in details but all deal in one way or another with mode or process of broad range of thinking skills and habits which is integral to mathematical problem solving. Overall, reasoning focuses on reflective processes whereby students build and extend mathematical knowledge.

Reasoning provides a strategy for students to engage in the activities of mathematical disciplines as mathematical knowledge is not immediately and inevitably comes from a set of axioms. Lakatos (1976) argued that knowledg**e** in mathematics comes from an iterative process of making conjectures, proving conjectures, finding counterexamples, and re-examining proofs. In the same vein, Stylianides (2008) explains that reasoning involves a broad range of skills and habits such as identifying patterns, making conjectures, and providing non-proof arguments. Accordingly, in explaining how reasoning can become a regular feature of mathematics, NCTM (2009) has highlighted four specific categories of habits that should become routine across different content areas, classrooms, and schools such as analysing a problem, implementing a strategy, seeking and using connections, and reflecting on a solution to a problem. These perspectives foreground reasoning is an integral aspect of all mathematical disciplines, specifically in mathematical problem solving.

2.3 Reasoning as Specific and General Skills in Mathematical Problem Solving

While reasoning has been acknowledged to be important in mathematical learning and problem solving, the relationship between reasoning and the domain in which reasoning is activated has raised questions about the specific and general nature of learning. Arguments have been made to support the claim that reasoning skills are general in nature. For example, prior to solving algebra word problems, a student decides that there are certain general problem-solving procedures that she has used with word problems in the past that enhanced her performance. Those procedures that she plans to employ include converting the world problem to a symbolic form, categorizing it according to solution method, and checking and verifying each answer. What this student has done is to engage in strategic processing of a more general nature. Similarly, while completing her algebra task, she might finds that an answer to a problem does not match the one in the back of the book. At this point, she decides to regroup her variables, simplify her equation, and try a different algorithm. She is again engaging in strategic behavior which required knowledge in much of more domain-specific nature. Both these strand of knowledge interact and complement each other and plays a prominent role in mathematics understanding and problem solving (Chinnappan & Lawson, 1996).

An alternative view of reasoning is that these processes are activated within the domain such as geometry (Chinnappan, Ekanayake & Brown, 2011). In their study, students activated reasoning that involved making and testing predictions, conjectures or hypotheses. In other problem solving contexts reasoning has been found to facilitate searching for patterns, explaining and justifying solutions.

Although there are competing views about where and when reasoning can be activated, the general nature of reasoning skills appear to support students access their prior knowledge and use that knowledge in addressing novel problems. In order to address this issue, we need to reconceptualise reasoning in terms of actions and effect of actions on activation and use of prior knowledge. One such model of reasoning that is consistent with this view is that provided by SRS.

2.4 Scientific Reasoning Skills

The range of SRS that students bring to learning and problem solving can be expected to assist them making progress in multiple ways. In this sense, SRS "encompasses the reasoning and problem-solving skills involved in generating, testing and revising hypotheses or theories, and in the case of fully developed skills, reflecting on the process of knowledge acquisition and knowledge change that result from such inquiry activities" (Morris, Croker, Masnick & Zimmerman, 2012, p. 65). Scientific reasoning differs from other skills in that it requires additional cognitive resources as well as an integration of cultural tools. Further, scientific reasoning emerges from the interaction between internal factors (e.g., cognitive and metacognitive development) and cultural and contextual factors. According to Lawson (2004), scientific reasoning pattern is defined as a mental strategy, plan, or rule used to process information and derive conclusions that go beyond direct experience. Hand, Prain and Yore (2001) argued that scientific reasoning abilities and habits of mind lie at the heart of scientific literacy, which involves the abilities and habits of mind to construct understanding of problems at hand, understanding the central concepts and unifying theories of science, and the ability to communicate and persuade others to take action related to those concepts and theories. Thus, this skill is related to cognitive abilities such as critical thinking and reasoning and can be described as processes of producing knowledge through evidence–based reasoning.

2.5 SRS and Knowledge Transfer in Mathematical Performance

The Science, Technology, Engineering and Mathematics [STEM] education community considers that transferable general abilities such as reasoning are at least as important for students to learn as is the STEM content knowledge (Chen & Klahr, 1999; Kuhn & Dean, 2004). This view is consistent with our assumption that reasoning skills act as a layer of general skills that assist students in the use of content knowledge.

As stated in the section of SRS above, Hand, Prain and Yore (2001) describe that scientific reasoning involves the abilities to construct understanding of the content knowledge and to give justification on actions that individuals have adopted. As such, scientific reasoning pattern plays an important role informing habit of minds as suggested by the NCTM (2009). Thus, our expectation is that students' scientific reasoning abilities could be utilised when solving complex mathematical problems. Accordingly, higher scientific reasoning levels among students could be expected to enhance mathematical problem solving performance.

3. Conceptual Framework

In this section, we provide a conceptual framework for this study. The Lawson's Classroom Test of Scientific Reasoning provided the framework to guide the analysis and interpretation of data.

3.1 Lawson's Classroom Test of Scientific Reasoning

In this study, we focus on a set of domain-general reasoning skills that are commonly needed for students to make progress to the mathematical problems which includes exploring a problem, formulating and testing hypotheses, manipulating and isolating variables, and observing and evaluating the consequences. Lawson's Classroom Test of Scientific Reasoning [LCTSR] provides a theoretical lense for assessing a range of SRS (Lawson, 1978, 2000). The test was designed to examine a small set of general reasoning ability dimensions which are crucial for the solution of problems in STEM including conservation of matter and volume, proportional reasoning, control of variables, probability reasoning, correlation reasoning, and hypothetical-deductive reasoning. The validity of the LCTSR had been established by several studies (e.g. Lawson, Bank & Lovgin, 2007; Bao, Fang, Cai, Wang, Yang, Cui, Han, Ding & Luo, 2009; Thoron & Myers, 2012).

There are three levels of reasoning according to LCTSR: *concrete, transitional* and *formal* operational reasoning. The *concrete* operational reasoning refers to thinking pattern that enable one to understand concepts and statements that make a direct reference to familiar actions and observable objects, and can be explained in terms of simple associations (for example, all squares are rectangles but not all rectangles are squares). Individuals formation at this level of reasoning are also able to follow step-by-step instructions, provided each step is completely specified (for example, solving two linear equations). Individuals also are able to relate his/her viewpoint to that of another in familiar situations (for example, students respond to difficult mathematical problems by applying a related correct rule). In this stage, individuals are unconscious of his/her own reasoning patterns, inconsistencies among various statements he/she makes, or contradictions with other known facts.

In contrast to *concrete* reasoning, *formal* reasoning patterns enable individuals to construct possible explanations as a starting point for reasoning about a causal situation. They can reason in a deductive manner to test their hypotheses. In other words, they can postulate causal factors, deduce the consequences of these possibilities and then empirically verify which of those consequences, in fact, occurs. Lawson (1978) categorized students at this stage as 'reflective thinkers'. For example, in solving mathematical problem, students' reasoning can be initiated with development of representations, used symbols and planning a course of actions. A critical aspect of this sequence of events is the activation of SRS at each of the phases.

The *transitional* operational stage is where individuals remain confined to *concrete* thinking or are only capable of partial *formal* reasoning. For example, proportional reasoning is the ability to compare ratios or make a statement of equality between two ratios. At *concrete* operational stage, students are not aware of ratio dependence and seek solutions by guessing. At the *transitional* stage, students are aware of objective dependence. Students seek solutions by estimation and later calculation, but assume that the change in one quantity produces the same change in the other quantity. In the *formal* stage, proportionality is discovered and applied to obtain correct solutions.

4. Purpose of the Study

We have argued that solving mathematical problems is a complex process that is influenced by various factors including reasoning. However, review of literature indicates that the nature of reasoning that supports problem solving is somewhat unclear. One school of thought argues that reasoning processes are located within maths. The alternative view is that reasoning skills are general in nature and are transferable across domain. The present study is based in the latter assumption that reasoning are general skills that learners need to activate during the course of mathematical problem solving regardless of the particular area of mathematics. These set of reasoning skills that we focus in the present study is SRS.

The purpose of the study was to identify the level of SRS attain by a cohort of upper Malaysian secondary school students (form four students, 16-17-year-old). In addition, we were interested to examine the impact of Achievement Group and Gender on levels of SRS and Mathematics Test. We sought data relevant to the following four research questions:

1. What are the levels of SRS among upper secondary school students?
2. Is there a relationship between SRS and Mathematics Achievement?
3. How does Achievement Group affect students' SRS and Mathematics Achievement?
4. How does Gender affect students' SRS and Mathematics Achievement?

5. Methodology

5.1 Design

This study employed a descriptive research design as our interest to generate information about the current status of the relationship between independent and dependant variables. Students responded to two test dependent variables. In addition, the correlational exploratory design was used to explore "the extents to which two or more variables co-vary, that is, where changes in one variable are reflected in changes in the other" (Creswell, 2008, p. 358) which was relevant to research question no. 2. The two key dependent variables are measures of SRS and Mathematics Achievement.

5.2 Participants

A total of 351 students from 14 Malaysian secondary schools participated in the present study. Participants in this study were upper secondary school students or Year 11 students (16-17-year-old). A stratified sampling technique was chosen as the population in this study was not heterogeneous. The technique is appropriate in order to obtain selection representative sample from each stratum reflecting Achievement Group and Gender (Wiersma & Jurs, 2005). The validity of stratified sampling could also provide a more accurate sample because the sample size distributions are quite similar within each stratum (Bryman & Cramer, 2005). Moreover, the advantage of stratified random sampling procedure was reduced sampling error.

All secondary students in Malaysia were required to complete a common mathematics curriculum in the first three years of high school. At the end of the three years all students were required to sit for a centralized examination, Lower Secondary Evaluation Examination (LSEE). Participants were assigned to two groups on the basis of their performance in mathematics and science in LSEE. Those students who obtained Grades A or B in mathematics and science in this examination were assigned to the High-Achievement group and their counterparts who obtained Grades C or D were the Low-Achievement group. Table 1 presents the distribution of participants based on Achievement Group and Gender.

TABLE 1 Distribution of participants based on Achievement Group and Gender

Achievement Group	Gender	Number of student	Percentage (%)
High	Male	42	12
	Female	56	16
Low	Male	134	38.2
	Female	119	33.9
Total		351	100

5.3 Tasks

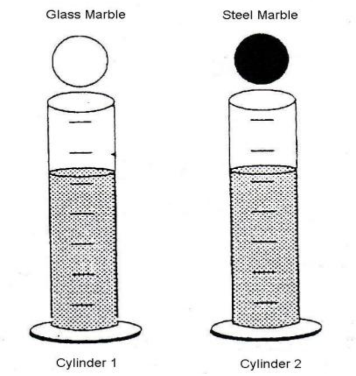
This section provides details of tasks that were used in this study. As discussed earlier, there were two tests used in order to generate scores for two dependent variables:
Test 1 - Scientific Reasoning Test (SRT).
Test 2 - Mathematics Test (MT).

Test 1 - Scientific Reasoning Test (SRT)

The SRT was used to measure the students' level of SRS. It has been adapted from Nor'ain, Norashiqin and Amalina (2012) which based on LCTSR (Lawson, 2000). This test was validated by mathematics education experts and had a high Kuder-Richardson 20 internal-consistency reliability coefficient index value of0.856. The test consisted of 12 pair items and was designed in a 'two-stage' multiple-choice format to illustrate problem scenarios. With each scenario, the first question focuses on the scenario content specifically, while the second question asks for reason the first answer is correct. Each answer for the first question has a corresponding reason in the second question.

Figure 1 shows an example of the item for SRT. This example evaluates the conservation of volume reasoning skills. Firstly, students have to think based on their experience or previous knowledge or facts where the water will rise when the glass marble is put into cylinder. Then, students have to make justifications why the water rose at that level. This involves individual applying the conservation reasoning to perceptible objects and properties namely 'remains the same' – is 'conserved' and thus since the two marbles have the same volume they will displace the same amount of water. Making prediction and giving explicit explanation are keys to successful completeness of this item. Both prediction and explanation are also important process of mathematical problem solving. Thus, we argue that these reasoning skills would contribute to the solution outcome of mathematical problems.

3a) To the right are drawings of two cylinders filled to the same level with water. The cylinders are identical in size and shape. Also shown at the right are two marbles, one glass and one steel. The marbles are the same size but the steel one is much heavier than the glass one. When the glass marble is put into Cylinder 1 it sinks to the bottom and the water level rises to the 6th mark.



If we put the steel marble into Cylinder 2, the water will rise

a. to the same level as it did in Cylinder 1
b. to a higher level than it did in Cylinder 1
c. to a lower level than it did in Cylinder 1

3b) because

a. the steel marble will sink faster.
b. the marbles are made of different materials.
c. the steel marble is heavier than the glass marble.
d. the glass marble creates less pressure.
e. the marbles are the same size.

FIGURE 1 Example of item in SRT

Scoring Rubric for SRT

As described earlier, SRT consisted of 12 questions in pairs and the respondents were required to select one correct answer and provided an explanation for the answer based on the list of alternatives answers provided. Scores will be given a value of 1 only if both the answer to every question is correct. If the answer either or both of the pairs are not correct, then no points will be given. Interpretation of scores is shown in Table 2. The SRT is considered a reliable and valid instrument that measures levels of *concrete–formal* operational the same as Piagetian thinking skills in secondary and college–age students (Lawson, 1978). The range of scores of SRS level is 0-12 which decomposes into three levels as suggested by Lawson.

TABLE 2 Scoring scheme for level of SRS

SRS Scores	Level of SRS
0 – 4	*Concrete*
5 – 8	*Transitional*
9 – 12	*Formal*

Students' scoring in the range of 0-4 on the test were categorised as *concrete* reasoners while students scoring in the range of 9-12 in the test were categorised as *formal* reasoners. Students' scoring in the range of 5-8 were categorised as *transitional* reasoners.

In Table 3, we defined each of SRS level and provide instances of activity for each of the three levels in relation to Item 18 of MT. Students were categorised as having *concrete* reasoning level if they need reference to familiar actions, objects, and descriptive properties; their reasoning is initiated with observations; needs step-by-step instructions; unconscious of his/her own reasoning patterns. For this item, students may only show the understanding concepts of perimeter and area of a rectangle and a circle in solving the problem.

Students were categorised as having *formal* reasoning level if they can initiated with imagined possibilities; uses symbols to express ideas; plan a lengthy procedure given certain overall goals and resources; conscious and critical of his/her own reasoning patterns. In this case, students may systematically plan to find perimeter of irregular shaded region in Diagram 11. This will involve finding the curve length of a semicircle and a quadrant using the formulae for the area and using the given information of the radius length.

Students were categorised as having a *transitional* reasoning level if they remain confined to concrete thinking or are only capable of partial formal reasoning such as they only understand and applying or reconceptualising the concepts of perimeter and area of a rectangle and area of a circle in a new context.

TABLE 3 Characteristics of level of SRS

Level of SRS	Interpretation	Example of Type of Reasoning as in Item 18 (MT)
Concrete	Needs reference to familiar actions, objects, and descriptive properties; Reasoning is initiated with observations; Needs step-by-step instructions; unconscious of his/her own reasoning patterns.	Understand concepts of perimeter and area of a rectangle and a circle.
Transitional	Remain confined to concrete thinking or are only capable of partial formal reasoning	Understand, applying or reconceptualising concepts of perimeter, area of a rectangle and area of a circle in a new context.
Formal	Initiated with imagined possibilities; uses symbols to express ideas; plan a lengthy procedure given certain overall goals and resources; conscious and critical of his/her own reasoning patterns	Systematically plan to find perimeter of irregular shaded region in Diagram 11. This will involve finding the curve length of a semicircle and a quadrant using the formulae for the area and using the given information of the radius length.

Test 2 - Mathematics Test (MT)

The MT was designed to measure students' Mathematics Achievement. The test was prepared by a panel of experienced mathematics educators, experienced teachers and mathematics curriculum experts. Bloom taxonomy was used in developing of the items for the tests as it allowed us in examining key concepts and using these concepts to solve the problems. This permitted us in explaining the use of three levels of reasoning skills. The items for the test were selected from a pool of resources such as textbooks, reference books and examination papers. The test consisted of 40 multiple choice questions that covered mathematical strand in the Malaysian Mathematics Syllabus (Year 8 – Year 11). This test was validated by mathematics education experts and had a high reliability index value of 0.895. The following is Item 18 of the MT. The three levels of SRS as played out within Item 18 are explained in column 3, Table 3. This is to ensure that one or more levels of SRS are activated by the students during solving mathematical problems.

FIGURE 2 An example of MT item

18. Diagram 11 shows a semicircle with a radius of 3 cm and a quadrant with a radius of 1 cm
inside a rectangle.

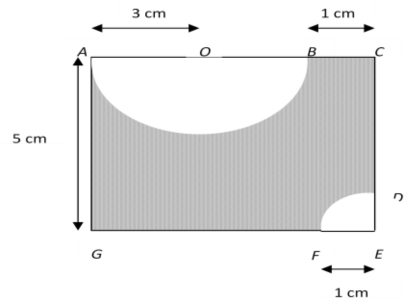


Diagram 11

Calculate the perimeter of the shaded region, in cm. (Use $\pi = \frac{22}{7}$)

A 16
B 18
C 21
D 27

Scoring Rubric for MT

Items in the MT were scored as follows:
1 - correct response.
0 - incorrect response.

5.4 Procedures

There were three phases in the study. The first phase was concerned with the development and fine-tuning of mathematics test. The details are explained in the MT task section. During the second phase, we pilot tested both the tests to allow for familiarisation process of the data collection processes, to validate the instruments used in the study and to establish the reliable measurements. The third phase involved the administration of the two tests. Both tests were administered during regular mathematics classes. Researchers and classroom teacher assisted in the administration of the tests. The MT, a one-hour paper and pencil format test, was administered in the first instance. Students were invited to complete the SRT in the following week. They were allowed a maximum of 40 minutes for SRT.

Prior to phases two and three, permissions were obtained from the Educational Planning and Research Department (EPRD) of the Ministry of Education, Malaysia and the Penang State Education Department to carry out the research. In addition, school administrations and parents were informed about the study and ethical clearances were obtained from parents, participating students and school principals.

5.5 Data Analysis

Statistical Analyses

Descriptive statistics such as frequency and percentage were used to describe the background information about the respondents who participated in this study. Means and standard deviation were used to describe the level of students' SRS and achievement level. In addition, there were two applications of inferential statistics used in this study, namely the t-test and Pearson correlation test. Exploratory data analysis was conducted for all the data collected in the study. In the analysis involving t-test, all the data met the assumptions namely the scale measurement at the interval level, random sampling, normally distributed, independence of groups and homogeneity of variance. For Pearson correlation analysis, the data met the assumptions such as both variables measured at the interval level, have linear relationship between the two variables, normally distributed, independence of groups and homogeneity of variance**.** These analyses ensure the accuracy of the tests' interpretation (Coakes, Steed, & Ong, 2009).

In this study, the independent t-test was used to answer the third and fourth research questions of identifying whether there was a difference in mean overall of SRS on Achievement Group and Gender. The Pearson correlation coefficient is a measure of the strength and direction of the linear relationship between two variables (Pallant, 2007). Pearson correlation coefficient is usually a decimal figure between −1 and 1. Higher values of correlation coefficients, regardless of their sign, indicate more strong relationships, whereas values closer to 0 from either side indicate weaker relationships. When the two variables are not related, their correlation coefficient is zero. In this study, Pearson correlation test is used to answer the second research question in determining the relationship between the SRS and Mathematics Achievement.

6. RESULTS

Four research questions were of interest to the present study. Data relevant to these research questions are presented below.

Research Question 1: What are the levels of SRS among upper secondary school students?

Table 4 shows the overall level of SRS exhibited by the participating Year 11 students. The findings of the study showed that 330 (94%) of the students achieved Level 1 (*concrete*) of SRS, 20 (5.7%) achieved Level 2 (*transitional*) and only 1 (0.3%) of them achieved Level 3 (*formal*). The overall mean level of the SRS was 1.76. This indicates that majority of the participating students exhibited *concrete* reasoning level of SRS.

TABLE 4 Level of SRS

SRS Level	N	Percentage(%)	SRS Mean Score	Standard Deviation
Concrete	330	94.0	1.50	1.18
Transitional	20	5.70	5.65	0.81
Formal	1	0.30	9.00	-
Total	351	100.00	1.76	1.55

Research Question 2: Is there a relationship between the SRS and Mathematics Achievement?
Table 5 displays the results of the analysis of correlation between the SRS scores and Mathematics Achievement. Overall, the correlation between the SRS and Mathematics Achievement was significant indicating a positive relationship between the two variables [(r = 0.593), p<0.05]. The coefficient of correlation (r = 0.593) indicating that there was a moderate positive relationship between the SRS and Mathematics Achievement. This suggests that if a student had a high score in SRT, he/she are expected to achieve high score in MT.

TABLE 5 Correlation between SRS and Mathematics Achievement

		SRS	Mathematics Achievement
SRS	Mean	1.76	56.40
	Pearson Correlation	1	.59**
	Sig. (2-tailed)		.000

****** sig. at p<0.01

Research Question 3: How does Achievement Group affect Students' SRS and Mathematics Achievement?
T-test analysis was performed to compare the mean scores of the overall level of SRS for Achievement Group. Analysis as presented in Table 6 showed there were differences in mean overall SRS between High and Low-Achievement groups [t (349) = 9.260, p < 0.05]. The mean SRS level for High-Achievement group (mean = 2.99) is better than the Low-Achievement group (mean = 1.28). However, based on the interpretation of the SRS scores as presented in Table 4, the level of SRS for both groups of students are in *concrete* reasoning level. The results also showed there were differences in mean overall MT score between High and Low-Achievement group [t(349) = 16.789, p < 0.05]. The mean MT score for High-Achievement group (mean = 81.02) is better than the Low-Achievement group (mean = 46.86). This indicates that students have different levels of SRS (in terms of their SRS score) and Mathematics Achievement as measured by the MT based on Achievement Group. Specifically the High-Achievement group had better SRS and MT scores than the Low-Achievement group.

TABLE 6 SRS score and MT score Vs Achievement Group

		Achievement Group			
Dependent Variable		High	Low	t-value	p-value
SRS score	Mean	2.99	1.28	9.260	**
	SD	1.66	1.21		
MT score	Mean	81.02	46.86	16.789	**
	SD	13.42	18.32		

** sig. at p<0.01

Research Question 4: *How does Gender affect Students' SRS and Mathematics Achievement?*
The results of t-test analysis as illustrated in Table 7 showed there was no significant difference in the overall level of SRS scores between boys and girls [t(349) = 0.765, p > 0.05]. Inspection of the mean value as a whole, boys (mean = 1.82) and girls (mean = 1.70) had quite a similar mean values.. In addition, analysis also revealed there was no significant difference in the overall MT scores between boys and girls [t(349) = 1.99, p = 0.05]. Both boys and girls had quite a similar mean values, boys (mean = 53.98) and girls (mean = 58.83).

These findings indicate that students have similar levels of SRS and Mathematics Achievement based on Gender.

TABLE 7 SRS score and MT score Vs Gender

Dependent Variable		Gender		t-value	p-value
		Boys	Girls		
SRS score	Mean	1.82	1.70	0.76	0.44
	SD	1.72	1.36		
MT score	Mean	53.98	58.83	1.99	0.05
	SD	23.49	22.22		

7. DISCUSSION

The study was designed to generate data relevant to issues about level of SRS and Mathematics Achievement among Malaysian students. The first research question addressed the level of SRS among upper secondary school students. We found participants in this study did not do well in SRT. Contrary to our expectation, there was not even spread in the scores for SRS. Almost all the students (94.0%) were in the *concrete* reasoning level and others were in the *transitional* (5.7%) and *formal* (0.3%) reasoning levels.

The second research question addressed the relationship between SRS and Mathematics Achievement. The results indicate that there was a moderate positive correlation between the SRS and Mathematics Achievement as measured by the MT. Data analysis relevant to Research Question 3 showed that students in the High-Achievement group performed significantly better than their Low-Achievement peers in the MT and SRS scores. Given the positive correlation betweeen SRS and MT, it can be argued that the higher MT scores of High- Achievement group can be atrributed to their superior SRS. However, the SRS scores for all the students including the High-Achievement group was lower suggesting they were operating at *concrete* level.

Interestingly, the relatively higher SRS scores for the High-Achievement group is still low in terms of the SRS level they have achieved. The mean SRS score for this group was 2.99 which falls well into the *concrete* reasoning level. However, as shown in Table 6, despite relatively low SRT scores for High- Achievement group, the score on MT for this group was significantly high (mean = 81.02) in comparison to the Low-Achievement group. The breakdown of SRS levels along Achievement Group did not generate significant differences. This is somewhat surprising given that the low SRS scores did not, as expected, lead to low MT scores. Thus, it would seem that students in High-Achievement group were using processes in addition to SRS, a claim that needs further investigation. The relationship between SRS and Mathematics Achievement for the Low-Achievement group was as per our expectation. That is, low SRS scores for students in the Low-Achievement group led to low score in MT. Thus, it would seem that for Low-Achievement group, SRS play a significant role in their solution search. These results are consistent with the findings of Chinnappan and Lawson (1996) who reported that general problem-solving strategies such as SRS would have a greater effect on low-achieving students.

Gender related analyses of SRS and Mathematics Achievement is relevant to the fourth research question. The result does not indicate significant differences for both SRS and Mathematics Achievement for Gender. This is in contrast to the findings as reported by MOE (2013) which indicates significant differences among girls and boys; girls are performed better than the boys. Thus further investigation is required to explain the pattern of results reported by MOE and these results of present study.

In our analysis of level of SRS, Mathematics Achievement and Gender related inputs, the investigation did not consider the cultural context of the participating students. The students in this study had three types of linguistic backgrounds- Malay, Mandarin and Tamil. It would be interesting to explore the link between students' linguistic background, reasoning skills and mathematical problem solving outcomes.

Review of literature has shown the existence of a positive link between SRS and achievement across different subject domains in secondary curriculum. For example, Schen (2007) found that students who have higher level of SRS tended to perform better in their study of Biology. Such studies that provide empirical evidence for the relationship between SRS and academic achievements are limited, particularly research linking reasoning with achievement in mathematics. In this study, we generated empirical data between SRS and Mathematics Achievement.

In the present study, we drew on Lawson's work concerning the three levels on the assumption that this level would capture the multitude of students' reasoning activated during novel mathematical problem solving. As mentioned above, although all students were operating at Level 1 of SRS, their performance in MT, particularly for the High-Achievement group, was high. It would seem that students are engaging in substantial reasoning when they complete the MT tasks. But this range of reasoning skill was not fully captured by Lawson's levels. We, therefore suggest that Lawson's level of reasoning need to be further developed in order to sensitize it to the nuances of mathematical thinking.

One could provide an alternative interpretation for the apparent lack of link between the low SRS scores for the High-Achievement group and their high MT score. It is plausible that there could be other processes operating in tandem with SRS during the solution of mathematical problem solving such as students' learning style. Equally, as suggested by Ball and Bass (2003) and Lakatos (1976) reasoning is embedded in deeper mathematical understandings. Thus, students's learning style and mathematical understandings constitute significant areas for future investigations..

7. Conclusions and Implications

The low levels of SRS as exhibited particularly by the students in the Low-Achievement group can be attributed to a number of factors. Firstly, it is possible that students were not given explicit instructions about reasoning and the role of reasoning in solving complex mathematical problems. Secondly, teachers may not understand the nature of deep reasoning and its role in helping students to solve non-routine mahematical problem solving. Based on the above assumptions, we conclude that teachers' professional programme in Malaysia must provide explicit instruction in reasoning skills.

Regular mathematics classrooms should allocate time to support students to reason without any constraints to produce correct or incorrect answers to predetermined outcomes. The current reform initiated by the Malaysian government in promoting higher order thinking skills is not grounded in a complete undesrtanding of what these skills are and how they are played out in mathematical prolem solving. Further research should be conducted to generate higher level of clarity about the roles of reasoning in mathematics learning.

Low performances in TIMSS and PISA by Malaysis students could be attributed to poor reasoning skills. This study indicates that reasoning skills are important at least for the Low-Achievement group. The results provided added support for the claim that in order to do well in TIMSS and PISA, more effort has to be invested and students need to be scaffolded in development of reasoning skills.

Acknowledgment
Manuscript preparation has been supported by Sultan Idris Education University, Malaysia as well as University of South Australia, Adelaide. We appreciate the participating schools and students for their cooperation and contribution to this study.

REFERENCES

Adel, P. & Shayer, M. 1994. *Really raising standards: Cognitive intervention and academic achievement*, London, UK: Routledge.

Ball, D. L. & Bass, H. 2003. Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 27–44). Reston, VA: National Council of Teachers of Mathematics.

Bao, L., Fang, K., Cai, T., Wang, J., Yang, L., Cui, L., Han, J., Ding, L. & Luo, J. 2009. Learning of content knowledge and development of scientific reasoning ability: A cross culture comparison. *American Journal of Physics, 77(12),* 1118–1123.

Barkl, S., Porter, A. & Ginns, P. 2012. Cognitive training for children: effects on inductive reasoning, deductive reasoning, and mathematics achievement in an Australian school setting. *Psychology in the Schools, 49(9),* 828–842.

Bieda, K. N., Ji, X., & Drwencke, J. & Picard, A. 2013. Reasoning-and-proving opportunities in elementary mathematics textbooks. *Journal for Research in Mathematics Education*. Retrieved 5 January 2013 from http://dx.doi.org/10.1016/j.ijer.2013.06.005.

Bryman, A. & Cramer, D. 2005. *Quantitative data analysis with SPSS 12 and 13: A Guide for social scientists*, London, UK: Routledge.

Chen, Z. & Klahr, D. 1999. All other things being equal: Children's acquisition of the control of variables strategy. *Child Development, 70,* 1098–1120.

Chinnappan, M., Ekanayake, M. B. & Brown, C. 2011. Specific and general knowledge in geometric proof development , *South Asian Association for Regional Cooperation* (SAARC) *Journal of Educational Research, 8,* 1–28.

Chinnappan, M. & Lawson, M. J. 1996. The effect of training in use of executive strategies in geometry problem solving, *Learning and Instruction, 6(1),* 1–17.

Cifarelli, V. 1999, Abductive inference: connections between problem posing and solving, In O. Zaslavsky (Ed.), *Proceedings of the 23rd annual conference International Group for the Psychology of Mathematics Education*, Haifa, Israel (Vol. 2, pp. 217–224). Haifa, Israel: University of Technion.

Coakes, S. J., Steed, L. & Ong, C. 2009. *SPSS Version 16.0 for Windows: Analysis without anguish*, Milton, QLD: John Wiley & Sons.

Creswell, J. W. 2008. *Research design: Qualitative, quantitative, and mixed methods approaches*, Oaks, CA: SAGE Publications, Incorporated.

Garofalo, J. & Lester, F. K. 1985. Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16,*163–176.

Hand, B. M, Prain, V., & Yore, L. 2001. Sequential writing tasks influence on science learning. In P. Tynjala, L. Mason & K. Lonka (Eds.). *Writing as a Learning Tool* (pp. 105–129). Boston, MA: Kluwer Academic Press.

Klahr, D. 2000. *Exploring science: The cognition and development of discovery processes*. Cambridge, MA: MIT Press.

Kuhn, D, & Dean, D. 2004. Connecting scientific reasoning and causal inference. *Journal of Cognition & Development, 5,* 261–288.

Krejcie, R. V. & Morgan, D. W. 1970. Determining sample size for research activities. *Educational and Psychological Measurement, 30,* 607–610.

Lakatos, I. 1976. *Proofs and refutation,* Cambridge, UK: Cambridge University Press.

Lawson, A. E. 2004. The nature and development of scientific reasoning: A synthetic View. *International Journal of Science and Mathematics Education, 2,* 307–338.

Lawson, A. E. 1978. The development and validation of a classroom test of formal reasoning. *Journal of Research in Science Teaching, 15(1),* 11–24.

Lawson, A. E. 2000. *Classroom test of scientific reasoning (revised).* Retrieved 10 September 2013, from http://www.ncsu.edu/per/TestInfo.html.

Lawson, A. E., Banks, D. L. & Logvin, M. 2007. Self-efficacy, reasoning ability, and achievement in college biology. *Journal of Research in Science Teaching, 44(5),* 706–724.

Leighton, J. P. 2004. Defining and describing reason. In J. P. Leighton, & R. J. Sternberg (Eds.), *The nature of reasoning* (pp. 3–11). Cambridge: Cambridge University Press.

Lester, F. K. 2013. Thoughts about research on mathematical problem- solving instruction. In L. Santos-Trigo & L. Moreno-Armella, International Perspectives on Problem Solving Research in Mathematics Education, a special issue. *The Mathematics Enthusiast, 10(1&2),* 245–278.

Ministry of Education. 2013. *Malaysia Education Blueprint 2013-2025 (Preschoool to Post-Secondary Education*, Putrajaya, Malaysia: Kementerian Pendidikan Malaysia.

Morris, B. J., Croker, S., Masnick, A. M. & Zimmerman, C. 2012.The emergence of scientific reasoning. In H. Kloos, B. J. Morris and J. L. Amaral (Eds.), *Current Topics in Children's Learning and Cognition* (pp. 61–82). Rijeka, Croatia: Tech - Open Access Publisher. Retrieved 5 January 2013 from http://creativecommons.org/licenses/by/3.0.

Mullis, I. V. S., Martin, M. O., Foy, P. & Arora, A. 2012. *TIMSS 2011 International results in mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College. Reterieved 2 January 2013, from http://timssandpirls.bc.edu/timss2011/downloads/T11_IR_Mathematics_FullBook.pdf.

National Council of Teachers of Mathematics. 2009. *Focus in high school mathematics: reasoning and sense making*. Reston, VA: NCTM.

Nor'ain M. T., Norashiqin, M. I. & Amalina, I. 2012. Correlation between scientific reasoning skill level and lecturers' teaching style among Mathematics undergraduates. *Scottish Journal of Arts, Social Sciences and Scientific Studies*, *3(1),* 129-141.

Organisation for Economic Co-operation and Development (OECD). 2013. *PISA 2012 Results.* Retrieved 4 January 2013, from http://www.oecd.org/pisa/keyfindings/pisa-2012-results.htm.

Pallant, J. 2007. *SPSS survival manual: A step by step guide to data analysis using SPSS for Windows.* Maidenhead: Open University Press.

Polya, G. 1954. *Mathematics and plausible reasoning* (Volume 1, *Induction and analogy in mathematics;* Volume 2, *Patterns of plausible inference*). Princeton: Princeton University Press.

Santos-Trigo, L. & Moreno-Armella, L. 2013. International perspectives on problem solving research in mathematics education, a special issue. *The Mathematics Enthusiast, 10(1&2),* 3–8.

Schauble, L. 1990. Belief revision in children: The role of prior knowledge and strategies for generating evidence. *Journal of Experimental Child Psychology, 49*, 31–57.

Schen, M. S. 2007. *Scientific reasoning skills development in the introductory biology courses for undergraduates.* Unpublished doctoral dissertation. Columbus, OH: Ohio State University.

Schoenfeld, A. H. 1992. Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–371). New York, USA: Macmillan.

Schoenfeld, A. H. 2013. Reflections on problem solving theory and practice. In L. Santos-Trigo & L. Moreno-Armella, International Perspectives on Problem Solving Research in Mathematics Education, a special issue. *The Mathematics Enthusiast,* 10(1&2), 9–34.

Simon, H. A. 1996. The patterned matter that is mind. In D. M. Steier & T. M. Mitchell (Eds.), *Mind matters: A tribute to Allen Newell* (pp. 357–362). Mahwah, NJ: Erlbaum.

Sternberg, R. J. (1977). *Intelligence, information processing, and analogical reasoning: The componential analysis of human abilities*. Hillsdale, NJ: Erlbaum.

Stylianides, G. J. 2008. An analytic framework of reasoning-and-proving. *For the Learning of Mathematics, 28(1),* 9–16.

Stylianides, G. J., Stylianides, A. J., & Shilling-Traina, L. N. 2013. Prospective Teachers' Challenges in Teaching Reasoning-and-Proving. *International Journal of Science and Mathematics Education, 11*, 1463–1490.

Thoron, A. C., & Myers, B. E. 2012. Effects of inquiry–based agriscience instruction on student scientific reasoning. *Journal of Agricultural Education, 53(4),* 156–170.

Tularam, G. A. (2013). Mathematics in finance and economics: importance of teaching higher order mathematical thinking skills in finance. *e-Journal of Business Education & Scholarship of Teaching, 7(1),* 43-73.

Walker, M. 2011. *PISA 2009 Plus results performance of 15-years olds in reading, mathematics and science for 10 additional participants*. Melbourne: Australia Council for Educational Research (ACER). Retrieved 30 August 2012, from http://research.acer.edu.au/cgi/viewcontent.cgi?article=1000&context=pisa.

Wiersma, W., & Jurs, S. 2005. *Research Methods in Education: An Introduction* (8th ed.). Boston, USA: Allyn and Bacon.