Extension of Tabular Method for Integral

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Abstract

Typically in Calculus when "Tabular Method" is taught, it is always assumed that one of the two factors will eventually become zero after repeatedly taking the derivative. But if we do not assume this extra fact, still we can use the tabular method in a very meaningful way to compute many integrals encountered in a Calculus class. This paper describe this procedure with examples. Instructor may want to implement this in their Calculus courses.

Keywords: Tabular Method; Calculus; Integral; Techniques of Integral

1. Introduction:

When compute integral of the form, $\int f(x)g(x)\,dx$ where "integral by parts" can be used, typically we use u-v method(George B. Thomas, 2004). So for example we set:

$$u = f(x),\ \ dv = g(x)dx$$
$$du = f^{'}(x)dx, \qquad v = \int g(x)dx$$
$$\int f(x)g(x)\,dx = uv - \int vdu = f(x)\int g(x)\,dx - \int \{\int g(x)dx\}f^{'}(x)dx$$

We only teach tabular method when multiple use of "integral by parts" is needed in a single problem and one of the factor from that integrand will eventually become zero after taking the derivative repeatedly. So let us assume that $f(x)$ will eventually become zero after we take the derivative repeatedly. Say$f(x)^4 = 0$. Now the tabular method will looks like:(Stewart, 2012)

$f(x)$	$g(x)$
$f^{(1)}(x)$	$\int g(x)dx$
$f^{(2)}(x)$	$\int [\int g(x)dx]dx$
$f^{(3)}(x)$	$\int [\int [\int g(x)dx]dx]dx$
$f^{(4)}(x) = 0$	$\int [\int [\int [\int g(x)dx]dx]dx]\,dx$

$$\int f(x)g(x)\,dx$$
$$= f(x)\int g(x)\,dx - f^{(1)}(x)\int \left[\int g(x)dx\right]dx$$
$$+\ f^{(2)}(x)\int \left[\int \left[\int g(x)dx\right]dx\right]dx -\ f^{(3)}(x)\int [\int [\int [\int g(x)dx]dx]dx]\,dx$$

But there is no need to assume that $f(x)$ must become zero eventually after repeatedly taking the derivative. In this case all we need is to add an extra term at the end which will be the integral of the product of the last row.

2. Extension of the Tabular Method:

Now in the above table let us assume that $f^{(4)}(x)$ is not necessarily zero. Then the extended tabular method will give the following result:

$f(x)$	$g(x)$
$f^{(1)}(x)$	$\int g(x)dx$
$f^{(2)}(x)$	$\int [\int g(x)dx]dx$
$f^{(3)}(x)$	$\int [\int [\int g(x)dx]dx]dx$
$f^{(4)}(x)$	$\int [\int [\int [\int g(x)dx]dx]dx]\, dx$

$$\int f(x)g(x)\,dx$$
$$= f(x)\int g(x)\,dx - f^{(1)}(x)\int\left[\int g(x)dx\right]dx$$
$$+\ f^{(2)}(x)\int\left[\int\left[\int g(x)dx\right]dx\right]dx - f^{(3)}(x)\int\left[\int\left[\int\left[\int g(x)dx\right]dx\right]dx\right]dx$$
$$+\int f^{(4)}(x)\left[\int\left[\int\left[\int\left[\int g(x)dx\right]dx\right]dx\right]dx\right]\,dx$$

We can extend or shrink the tabular table as we please.

Example 1:

$$\int e^{ax} \sin(bx)\, dx$$

Here a and b are real numbers. We take $f(x) = e^{ax}$ and $g(x) = \sin(bx)$

e^{ax}	$\sin(bx)$
ae^{ax}	$-\frac{1}{b}cos(bx)$
a^2e^{ax}	$-\frac{1}{b^2}sin(bx)$

$$\int e^{ax} \sin(bx)\, dx$$

$$= e^{ax}\left\{-\frac{1}{b}cos(bx)\right\} - ae^{ax}\left\{-\frac{1}{b^2}sin(bx)\right\} + \int [a^2e^{ax}\{-\frac{1}{b^2}sin(bx)\}]\, dx$$

Move the circled term on the left hand side and solving for $\int e^{ax} \sin(bx)\, dx$ and simplify, we get:

$$\int e^{ax} \sin(bx)\, dx = \left(\frac{b}{a^2 + b^2}\right)\left[\frac{a}{b}e^{ax}\sin(bx) - e^{ax}\cos(bx)\right]$$

Which can be simplified to

$$\frac{e^{a\,x}\,(-b\cos(b\,x) + a\sin(b\,x))}{a^2 + b^2}$$

Example 2:

Derive the recursion formula for $\int sin^n(x)dx$.

We rewrite $\int sin^n(x)dx = \int sin^{n-1}(x)\sin(x)\,dx$

We choose $f(x) = sin^{n-1}(x)$ and $g(x) = \sin(x)$

Let us look into the extended tabular table in this case:

$sin^{n-1}(x)$	$\sin(x)$
$(n-1)sin^{n-2}(x)\cos(x)$	$-\cos(x)$
$(n-1)(n-2)sin^{n-3}(x)cos^2(x) - (n-1)sin^{n-1}(x)$	$-\sin(x)$

$$\int sin^n(x)dx = sin^{n-1}(x)\{-\cos(x)\} - (n-1)sin^{n-2}(x)\cos(x)\{-\sin(x)\} + \int [(n-1)(n-2)sin^{n-3}(x)cos^2(x) - (n-1)sin^{n-1}(x)][-\sin(x)]\,dx$$

Using the fact that $cos^2x = 1 - sin^2x$ and simplify and solve for $\int sin^n(x)dx$ we get the following familiar recursion formula:

$$n\int \sin^n x\,dx = -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx,$$

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

3. Conclusion:

We see that a very simple twist or extension of the tabular method can be very useful and practical. The u-v method can be completely avoided and can be replaced by this extended tabular method which is easy and fast. The instructors teaching calculus classes are encouraged to try this method in their teaching.

References

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