Application areas of Stochastic Dominance in Project Management

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Abstract

Stochastic dominance is a term describing a set of relations that holds between two distributions. A common application of stochastic dominance is in project management, which is the main focus of the paper. It is a fundamental concept in decision theory with uncertainty which describes when a particular random variable is preferred to another random variable based on preferences regarding the outcomes. Project management has technical, financial, scheduling and environmental risks, each of which can be modelled using stochastic dominance as outlined in this study. This article reviewed related literatures based on network scheduling, resources scheduling, time-cost tradeoffs, net present value, and project evaluation and review techniques, and show how stochastic dominance can be applied in them.

Key word: Stochastic dominance, Project management, Random variable, Distribution functions, Network techniques

1. Introduction

Stochastic dominance is an old principle to rank random variables, determine the efficient sets and facilitate the decision making process (Davidson, 2006). They indicate when one random variable ranks higher than the other by specifying a condition which the difference between their distribution functions must satisfy (Wolfstetter, 1996). In other words, a preceding activity is ranked higher than its succeeding activity. According to Shapiro and Philpott (Shapiro & Philpott, 2007) stochastic dominance is an approach for modelling optimization problems that involves uncertainty. It deals with situations where random variables are used to describe some or all the parameters (Ukwuoma, 2012). Let X be any random variable. Suppose $Y = X + 1$, then $X < Y$, therefore Y is dominant over X. If Y is a random variable independent of X and has the same distribution as $X + 1$, then events $X < Y$ and $X > Y$ can both happen with positive probability. The sources of random variables may be several depending on the nature and types of problem. Stochastic dominance treats problems where optimization and uncertainty appears together (Branda, 2012). Such problems arise in industry, engineering, finance, agriculture, logistics, even in project management (Chen, Sim, Sun, & Zhang, 2006). To determine whether a relation of stochastic dominance holds between a pair of distributions, the distributions are normally characterized by their cumulative distribution function. Under uncertainty, a decision problem is classified as choosing among a set of stochastic variables. These variables are interpreted as network of activities with distribution functions $F_i(x)$, $i = 1, 2, . . . , n$. The study of stochastic dominance is interested in what it contributes to the solution of complex decision problems and their comparative statics of risk. This paper will not give a detailed exposition of the theory, but only elaborate on a number of its applications in various areas of project management. Though there are many other examples, we will address the following issues:

- Will the network schedule increase its completion time if the random duration of each activity becomes larger, and what if it is subject to change?
- Consider the resources demanded by each activity in a project and allocation decision; will allocation of resources to an activity increase if the resource demand becomes more volatile?
- Considering the uncertainties in minimizing project cost and duration; is there arbitrary cost, and how does the cost probability change if more activities contemplate their durations?
- Can one unanimously distribute cash flow among activities to determine total project duration?
- If activities are considered as random variables with probability distributions, can the main task be to compute the marginal probability distribution of the project completion time.

The rest of the work is divided into sections with the related literature in the next section followed by methods applied in this research work. The findings and discussion follow thereafter and finally conclusion of the work is drawn.

2. Related Works

In project management, stochastic dominance can be applied to determine the completion time of a project using network diagram (Chen, Sim, & Sun, 2006). It can also be used to minimize the completion time of a project subject to the constraints that the total available resources does not exceed the available budget (Chen, Sim, Sun, & Zhang, 2006). A project can be informally defined for our purposes as a set of precedence related activities that have to be performed using diverse and usually limited resources. Resource scheduling is the part of project management that deals with deciding when to execute which activities and how to allocate resources to the activities (Leus, 2002). The objective function in stochastic scheduling is usually the expected value of a function of the activity durations. Limited resources may impose another precedence relationship between activities using the same resources and this may increase the project duration (Kurtulus & Narula, 1985). This limited resources in network scheduling is randomly selected based on the resource availability.

Stochastic dominance can be applied in project time-cost trade-off problems (Laslo, 2003). Feng et al (2000) note the difficulty in evaluating time-cost trade off problems using deterministic methods. The time-cost trade-off analysis assumes that time and cost of an option to complete an activity are deterministic(Feng, Liu, & Burns, 2000) but in reality the follow a certain probabilistic distribution as indicated by historical data, resulting in stochastic time-cost trade-off problems. Most researchers(Feng, Liu, & Burns, 2000; Cohen, Golany, & Shtub, 2007; Afshar & Kalhor, 2011) recognize that time-cost trade-off problems are stochastic in nature, most working solution assume that time and cost for completing an activity are deterministic (Akpan, 2001). Most literature on time-cost tradeoffs assume deterministic activity durations (Deckro, Herbert, Verdini, Grimstrud, & Venkateshwar, 1995; Laslo, 2003; Akpan E. , 2012), these durations are uncertain or random in nature, hence is a stochastic problems which Cohen et al (Cohen, Golany, & Shtub, 2007) call stochastic time-cost trade-off problems.

Net present value (NPV) is another area in which stochastic dominance can be applied in project management (Consuegra & Dimitrakopoulos, 2010). In software projects, a realistic schedule encompasses the time value of money. Russell(Russell, 1970)introduce net present value of money called payment scheduling problem using Taylor expansion of net present value in connection with linear dominance method. The project cash flows are uncertain and in order to address these uncertainties, Feng et al (Feng, Liu, & Burns, 2000) allocates a stochastic function to activity cost and time.

Project evaluation and review techniques (PERT) is a project management techniques useful in managerial functions of planning, scheduling, and controlling (Rao, 1977). It uses a stochastic method based entirely on beta distribution derived from three time estimates for each activity, which are optimistic time or probable earliest completion time, the most likely time or the most probable completion time, and the pessimistic time or the probable latest completion time (Eze, Obiegbu, & Jude-Eze, 2005). The distribution ranges from optimistic time to the pessimistic time with the most likely time as the mode of the distribution. The mean and variance are two statistics specified by beta-distribution in PERT. One important feature of PERT is that the probability rules can be applied to find the probability of completing a project by any given date.

3. Method

During the research process, the techniques in project management that entail random variables were considered. The procedure we survey here consists of the following steps. First, considering the activity network diagram and resources scheduling, then time-cost tradeoffs problem and net present value, finally project evaluation and review techniques. Each of these techniques was discussed showing how stochastic dominance can be applied in it. The strategy adopted does not warrant us to prove the findings in details, thus no example used to endorse our arguments.

4. Techniques where Stochastic Dominance is applicable

4.1 Stochastic Dominance of Network Scheduling

Project management problems can be represented by a directed graph or directed acyclic graph popularly known as the network diagram with nodes $N = \{1, 2, ..., n\}$ and arcs A contains in $\{(i,j): i,j = 1, 2, ..., n\}$ meaning that activity j cannot start until activity i completes(Kim, Boyd, Patil, & Horowitz, 2007). A stochastic activity network is an activity network with random activity durations. It was developed in the late 1950s to analysed project management and scheduling (Pich, Loch, & De Meyer, 2002). Hence an arc $(i,j) \in A$ is an activity that connects event i to event j. By convention, event 1 is used as starting node while event n is the ending node diagrammatically represented as

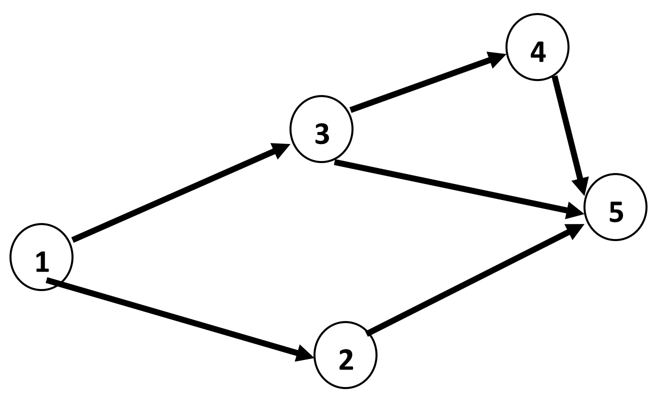

Fig 1: Network Diagram

Each activity has a random duration t_{ij} and the completion must adhere to precedence constraints where for each activity $k \in N$, we denote a set of its predecessor node as $Pred(k)$ and its successor node as $Succ(k)$, i.e. $Pred(k) = \{i \in N: (i, k) \in A\}$, $Succ(k) = \{j \in N: (j, k) \in A\}$ (Chen, Sim, & Sun, 2006; Kim, Boyd, Patil, & Horowitz, 2007). Starting nodes have no predecessors and ending nodes have no successors.

If activity e_1 precedes activity e_2 then e_1 must be completed before e_2 starts, therefore our random duration t_{ij} is independent of other activities. Suppose we have a path p of length k such that p =

$\{i_1, i_2, ..., i_n\}$ where $(i_j, i_{j+1}) \in A\ \forall\ j = 1, 2, ..., k-1$; therefore the length of path p is denoted as $l(p) = k$ and the total duration $d(p) = t_{i1} + t_{i2} + ... + t_{ik}$. Let y_i denote the completion time of activity i then the maximum duration of all paths finishing at i is $y_i = max\{d(p)$ given that p is a path finishing at $i\}$(Hoppe, 2007). We denote the maximum completion time of all activities in the network as $y_{max} = max(y_i)$ where i = 1 ,2, ..., n. when all quantities that depend on the activity durations are random variables, we have stochastic activity network. Taking the total completion time as the objective function, the network diagram has the optimization problem thus:

$$minimize\ y_{max}$$

$$Subject\ to\ x \in A$$

where A denotes the constraints set and x the design variables affecting the activity durations

4.2 Stochastic dominance in Resource Scheduling

In project planning problem, various resources can be used to complete project activities (Shankar, Raju, Srikanth, & Bindu, 2011). The resources used in a project are subject to varying demands and management need to know what activity and resources are critical to the project duration. The resource demand and allocation are random, varying from project to project or activity to activity even within the same project. This shows that the resources are uncertain but assumed to lie in some given set of possible values. Thus one might seek a solution that is feasible for all possible parameters and optimizes a given objective function. Suppose our c_{ij} denotes the cost of using each unit of resource for the activity on the arc (i, j) and t_{ij} depends on additional amount of resource$\lambda_{ij} \in [0, 1]$, our goal is to find a resource allocation to each activity $(i,j)\epsilon A$ that minimizes the total project cost while ensuring that the probability of completing the project within T days is at least $1 - \lambda$. There are multiple ways of selecting $\lambda_0$andλ_{ij}, $(I,j)\ \epsilon\ A$ so that the project will complete timely with probability of at least $1 - \lambda$. A reasonable way of selecting these values; λ_0, λ_{ij}, $(I,j)\ \epsilon\ A$ is to minimize the total budget of uncertainties for all the constraints as follows

$$Min\ \ \beta_0 + \lambda_0 \sum_{(i,j)\in A} \beta_{ij}\lambda_{ij}$$

Subject to

$$\lambda_0 + \sum_{(i,j)\epsilon A} \lambda_{ij} \leq \lambda$$

We then minimize the expected completion time of the project subject to the constraint that the total available resources is not more than the budget C. This project management problem (1) can be represented stochastically as:

$$\text{Minimize } y_{max}$$
$$\text{Subject to} \quad c\lambda \geq C$$
$$\lambda \geq 0$$

4.3 Stochastic Dominance of Time-Cost Tradeoffs

Recently, Project management has witnessed technological improvements providing managers with more options to choose from. Some of these technologies may intuitively allocate more resources to an activity, or shorten the duration with possible increase in direct cost resulting to tradeoff between time and cost.Time-cost tradeoff means crashing or shortening the duration of an activity by using additional resources, thus increasing its cost (Wiest & Levy, 1997). The additional cost is economically worthwhile and sometimes necessary to satisfy some constraints (e.g., to meet a project's deadline).The time cost trade-off problems are viewed as one of the most important aspects of project decision making. The stochastic nature of time and cost adds additional dimension of complexity to the already hard to solve combinatorial problem. In time-cost trade-off problem(Cohen, Golany, & Shtub, 2007)the duration of each activity is a function of the resources allocated to it(Deckro, Herbert, Verdini, Grimstrud, & Venkateshwar, 1995).A realistic schedule encompasses time value for money(Afshar & Kalhor, 2011).

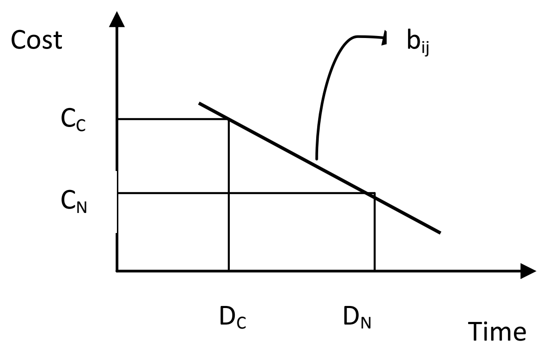

Fig 2 Time-Cost Linear Relationship (Eze, Obiegbu, & Jude-Eze, 2005)

Figure 2 assumes that (Ukwuoma, 2012), D_N = Normal time = m_{ij}
D_C = Crash Time = n_{ij} C_C = Crash Cost = a_{ij}
The direct cost is $C_{ij} = a_{ij} - b_{ij}t_{ij}$ where $m_{ij} \geq t_{ij} \geq n_{ij}$ and b_{ij} is the slope

$$c(t) = \Sigma c_{ij}(t_{ij})$$

The associated project duration is c_{max} with time related overhead cost C; thus the project cost is $F(t) = c(t) + C.c_{max}$ and the project completion date is formulated as

Minimize F(t) given $t_{ij} \in A$, $c_{max} \leq D$ where D is the project completion date
Subject to

$n_{ij} \geq t_{ij}$

$t_{ij} \geq m_{ij}$

Suppose $t_{ij} = T_{ij} – y_{ij}$ and assuming $c_{ij}$is function t_{ij}. Let x_i denote the starting time of node i = {1, 2, ..., n} and n is the finishing node thus if $x_1 = 0$, x_n is the project duration, P_j is a set of the predecessors of node j, $c_{N, ij}$ is the normal cost of activity {ij} associated with normal duration of T_{ij} and b_{ij} is the difference between the normal cost and the crash cost call the slope of crashing activity {ij} by a single time unit. For time-cost tradeoff analysis, we make the following assumption based on figure 1:(Cohen, Golany, & Shtub, 2007)

i) The normal duration for each activity {ij} is uncertain and bounded within a known interval T_{ij} (Deckro, Herbert, Verdini, Grimstrud, & Venkateshwar, 1995)

ii) The normal cost of an activity is an uncertain, non-decreasing function of its uncertain normal duration denoted as $c_{N,ij}$

iii) The slope b_{ij} is known and constant throughout every realisation of T_{ij}. For instance, consider the activity of coding in software development project. Suppose we crash this activity by allocating more man-hours to it, and its normal duration, $T_{ij} \in [-T_{ij}, +T_{ij}]$ is stochastic due to the possible needs to remove bugs detected during testing. This uncertainty does not affect the technology requirement of this activity nor does it change the way it is crashed, thus b_{ij} remains constant(Laslo, 2003)

Based on these assumptions, we represent the project time-cost tradeoff as stochastic dominance problem (Cohen, Golany, & Shtub, 2007):

Min $\Sigma(C_{N,ij} + b_{ij}y_{ij}) + Cx_m$

Subject to $X_j – x_i + y_{ij} \geq T_{ij}$

$y_{ij} \geq 0 \qquad \forall j, \forall i \in P_j$

$y_{ij} \leq T_{ij} - M_{ij} \qquad \forall_j \forall_i \in P_j$

$x_i = 0$

$x_m \leq D$

4.4 Stochastic Dominance of Net Present Value

The financial aspect of project management is analysed with discount methods assuming a dynamic variable value of money over time. Discounting deals with the computation of the present value of future cash flow based on the current value of money determined with the use of adopted rate of interest on capital (Klimek & Lebkowki, 2013). Net Present Value is appropriate to determine the riskiness of a project and advice the owner on whether the project is acceptable or not. Waligora (Waligora, 2008) says that net present value (NPV) reflects the change in the value of money in time and includes total cash flows related to the project. These cash flows are random and therefore a solution to such an issue is available through stochastic optimizer which has the ability to integrate uncertainty into the scheduling process using the predefined discount rate α

$$NPV = \sum_{i=1}^{n} \frac{A_i}{(1+\alpha)^{ti}}$$

Where n = total number of individual cash flows recorded within the period being analysed

A = the value of the i-th cash flows

t_i = the time of occurrence of the i-th cash flow, counted in capitalization periods for the given discount rateα. Mathematically, there is a set of activities denoted by $i \{i \epsilon A_p\}$ and corresponding to each activity, there is a set of implementation node denoted by $j \{j \epsilon M(i)\}$. t_{ij} and c_{ij} are time and cost of activity i when it is implemented by node j. The total project duration (T) and discounted cash flow (DC) can be minimized as defined in the equations below:

$$\min(T) = \max(T_p) = \sum_{i\,\epsilon\,Ap} \sum_{j=1}^{n_{M(i)}} x_{ij}\, t_{ij} \quad (2)$$

$$\min(DC) = \sum_{m=s}^{f} \frac{C_m}{(1+r)^m} = \sum_{m=s}^{f} \frac{\sum_{i\in A_m} \sum_{j=1}^{n_{M(j)}} x_{ij}\, c_{ij}\, pe_{i,m} + O_m}{(1+r)^m} \quad (3)$$

Where: T_p is the duration of path p in the activity network

A_p is the set of activities on path p in the activity network

$n_{M(i)}$ is the number of feasible operation nodes of activity i

x_{ij} is zero-one variable where $x_{ij} = \begin{cases} 1 \text{ if activity } i \text{ is implemented by node } j \\ 0 \quad \text{otherwise} \end{cases}$

s and f are start and finish nodes of a project, (s =1)

C_m is the total cost paid in m^{th} month

r is the interest rate

A_m is a set of activities which are entirely or partially pending in month m

$pe_{i,m}$ is the percentage of the duration of activity I pending in month m

O_m is the overhead cost of month m

The net present value increments in the order of 10 – 25% due to the use of stochastic optimizer, in such cases, development targets are met. The optimization process is based on the economic value of each task which belongs to the set of activities being scheduled. The expected value of an activity is calculated using its expected returns. The objective function involves the maximization of the expected net present value $E\{(NPV)_i^t\}$, which is generated by developing a phase at a given period. The objective function is

$$\text{Max}\sum_{i=1}^{n} \text{E}\{(\text{NPV})i^{t}\}b_{i}^{t} \qquad \text{(Consuegra \& Dimitrakopoulos, 2010)}$$

Where i = number of activity, t = time period, b_i^t = a variable representing the activity to be carried out at time t. The objective function is subject to analysis, design and processing constraints.

4.5 Stochastic Dominance in Program Evaluation and Review Technique (PERT)

A fundamental problem in PERT networks is to identify a project's critical paths and its critical activities (Dodin, 1984). A project is represented by a directed acyclic network where the nodes represent duration of activities and the arcs represent precedence constraints. In classical PERT, duration of activities is assumed to be known constants, and the task is to identify a critical path from start-time to finish-time such that the project completion time is the sum of the duration of the activities on the critical path. These activities are called *critical*, since a project could be delayed if these activities were not completed in the scheduled time. In stochastic PERT, activities are considered as random variables with probability distributions, and the main task is to compute the marginal probability distribution of the project completion time.

The problem of computing the marginal probability distribution of the project completion time is a difficult problem. Thus many approximate techniques have been developed. A classic solution proposed by Cinicioglu and Shenoy (Cinicioglu & Shenoy, 2006) is to assume that all activities are independent random variables and that each activity has an approximate beta distribution. The three time values estimated in this method for executing each activity are: most probable time of completion *m*; the time spent in most cases to execute an activity under similar conditions, the time with maximum frequency, minimum (optimistic) completion time *a*; the minimum time necessary to execute an activity, and maximum (pessimistic) completion time *b*, the maximum time necessary to execute and activity. The expected duration of each activity is then approximated by $\frac{(a+4m+b)}{6}$, and its variance is approximated by $\frac{(b-a)^2}{6^2}$. Using the expected duration times, the critical path is computed using the classical deterministic method. The mean and variance of the distribution of the project completion time is then approximated as the sum of the expected durations and the sum of variances of the activities on a critical path.

5. Discussion

Stochastic dominance has been applied in many field of research including project management both hypothetically and empirically. The motivation of applying the stochastic dominance rule in project management is more than the aforementioned 5-fold. The concept outlined above is mainly concerned with the application areas which is different from formulating stochastic models or proving them. There is a need to carryout in-depth study of each area mentioned in this study. Our results confirm that resource allocation is usually difficult even when objective-function evaluation by itself is not intractable, thus justifying solution approaches by means of implicit enumeration or integer programming techniques.

6. Conclusion

In this paper, the application of stochastic dominance in project management has been considered. Finally, the model could be applied in earned value analysis which standardizes contractor requirement for reporting cost and schedule performance on major projects.

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