Fractional-order Chaotic Filter with Generalized Memory

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Abstract

The paper proposes a non-traditional filter in the context of the fractional Poincare return time. The advantage of the filter is its integrative structure that characterizes the nonlinear properties of fractional order chaotic systems.

Keywords: fractional filter, Poincare return time, generalize memory, visualization.

1. Introduction

This article is a continuation of work on the extended clearly and fuzzy-reflexive filter, detailed in the [1]. Use axiomatic extended filters to successfully implement the tasks in the context of satisfaction the convergence of the solution.

Hammer P.S. [2] define a filter "as any device that receives or transmits certain elements in the set (system) and rejects others. …" So functions are filters, filters equation, system of axioms, relations are also considered the definition of filters [1].

As regards the concept of fractional chaotic filter here, of course, there are some differences from traditional designs in the theory of structures.

In the physicist of the open systems examples of filters can be fractional-order chaotic and hyperchaotic systems. Naturally, with such filters is very hard work. It is therefore proposed some non-traditional filter, which characterizes integrative properties systems.

In this context proposed filter, which is the Poincare return time of fractional order.

2. Notation and Preliminaries

2.1. n-dimensional-fractional-order chaotic system.

Consider the following n-dimensional-fractional-order chaotic system [3]

$$D^q X = F(X, X_0, \theta), \tag{1}$$

where $X = \left(x_1, x_2, \ldots, x_n\right)^T \in R^n$ denotes the n-dimensional state vector of the original system; X_0 - represents the system initial state, $q = (q_1, q_2, \ldots, q_n)^T \in R^n$ is a set of fractional order of the original system, and $\theta = (\theta_1, \theta_2, \ldots, \theta_D)^T \in R^D$ is the value of original system parameters.

Let the fractional-order derivative of the function $f(t)$ in the Caputo sense is defined as [3]:

$$D^q f(t) = J^{m-q} f^{(m)}(t). \tag{2}$$

Here, q is the fractional order, m is an integer that satisfies $m-1 \le q < m, f^m(t)$ is the ordinary mth derivative of f, and J^{μ} is the Riemann-Liouville integral operator of order $\mu > 0$, defined by

$$J^{\mu} g(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} g(\tau) d\tau, \tag{3}$$

where $\Gamma(\cdot)$ denotes the gamma function. A particularly important case in many engineering applications is $0 < q < 1$. In this situation, Eq. (2) together with Eq. (3)

$$D_*^q f(t) = \frac{1}{\Gamma(1-q)} \int_0^t (t-\tau)^{-q} f'(\tau) d\tau . \tag{4}$$

The operator D_*^q is often called "q th-order Caputo differential operator" and will be used throughout the paper.

2.2. Topological fractional-order space

Definition 1. The number is called as a metric order of a compact A

$$k = lim(-ln N_A(\varepsilon)/ln \varepsilon), \tag{5}$$

where ε - the sphere of radius ε; $N(\varepsilon)$ - number of spheres in a final subcovering of a set.

The lower bound of metric orders for all metrics of a compact A (called by metric dimension) is equal his Lebesgue to dimension.

However it appeared that the metric order entered in [4], coincides with the lower side the fractal dimension of Hausdorff-Bezikovich defined in the terms "box-counting".

Takes place

Theorem 1 [4]. For any compact metric space X .

$$dim\, X = inf \left\{ \underline{\underline{lim}}_{\varepsilon \to 0} \frac{log\, N_{\varepsilon,d}(X)}{-log\, \varepsilon} : d \ \ is\ a\, metric\ on\ X \right\},$$

where $N_{\varepsilon,d}(X) = min\{|U| : U\ is\ a\ \ finite\ open\ \ covering\ of\ \ X\ \ with\ mesh \le \varepsilon \}$.

From here (X, d_f) - compact fractal metric space with dimension d_f .

Here it is important to note that at the description of properties of systems with fractional structure it is impossible to use representation of Euclidean geometry. There is a need of the analysis of these processes for terms of geometry of fractional dimension.

Remark. In [5] presented results of communication of a fractional integrodifferentiation (in Riman-Liouville or Gryunvalda-Letnikov's terms) with Koch's curves.

It is noted that *biunique communication between fractals and fractional operators does not exist*: fractals can be generated and described without use of fractional operations, and defined the fractional operator not necessarily generates defined (unambiguously with it connected) fractal process or fractal variety.

However use of fractional operations allows to generate other fractal process (variety) which fractal dimension is connected with an indicator of a fractional integrodifferentiation a linear ratio on the basis of the set fractal process (variety).

In [6] fractional integrals of Riman-Liouville are understood as integrals on space of fractional dimension. Thus the indicator of integration is connected with dimension of space an unambiguous ratio.

In this regard consideration of dimension of chaotic systems of a fractional order causes interest. So, in [6] was noted that dimension of such systems can be defined by the sum of fractional exponents Σ, and $\Sigma < 3$ is the most effective.

Let the chaotic fractional system of Lorenz take place [6]:

$$\frac{d^{\alpha}}{dt^{\alpha}}x = \sigma(y-x)\ \frac{d^{\beta}}{dt^{\beta}}y = \rho x - y - xz'\ \frac{d^{\gamma}}{dt^{\gamma}}z = xy - bz\,. \tag{6}$$

Here $\sigma = 10,\ \rho = 28,\ b = 8/3;\ 0 < \alpha, \beta, \gamma \le 1,\ r \ge 1.$

Then fractional dimension of system of the equations (6) will have an appearance [6]:

$$\alpha + \beta + \gamma = \Sigma\,.$$

So, for example, for Lorentz's system with fractional exponents $\alpha = \beta = \gamma = 0.99$, effective dimension $\Sigma = 2.97$.

This, in the context of fractional dynamics let $\tilde{X}$ - any set of nonlinear physical systems, A^{α} - a subset of a set $\tilde{X}$ of systems of a fractional order with memory $A^{\alpha} \subset \tilde{X}$. Then a triad $(\tilde{X}, A^{\alpha}, \Sigma)$– compact fractional metric space with dimension Σ.

Let's designate $W \in (X, d_f)$. On the basis [7] and remarks $(\tilde{X}, A^{\alpha}, \Sigma) \subset W$.

Let's consider transformation W at an angle of communications of average time of return of Poincare $\langle\tau\rangle$ with d_f and "residual" memory $J(t)$.

Here $g : \langle\tau\rangle \Rightarrow d_f l : d_f \Rightarrow J(t) \chi : \langle\tau\rangle \Rightarrow (g, l)$.

From here $U \in (X, \langle\tau\rangle)$ - the generalized compact metric space of Poincare with dimension $\langle\tau\rangle$.

2.2. Fractional Lyapunov Exponent

In this subsection, we recall the notion of fractional Lyapunov exponent for an arbitrary function in [8]. Let $\|\cdot\|$ be an arbitrary but fixed norm on R^d.

Definition 2 [9]. The function $E_\alpha : C \to C$ which is defined by

$$E_\alpha(z) := \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(1+\alpha k)} \tag{7}$$

is called Mittag-Leffler function.

For any $\alpha \in (0,1)$, the restriction $E_\alpha : R \to R$ of the Mittag-Leffler function is strictly monotonically increasing. Thus, the restriction of the Mittag-Leffler function E_α on R is strictly monotonically increasing. Furthermore, using [10, Theorem 1.3, p.33], and [10, Theorem 1.4, p.33], we obtain that

$$\lim_{z\to\infty} E_\alpha(z) = \infty \quad \text{and} \quad \lim_{z\to\infty} E_\alpha(z) = 0. \tag{8}$$

Consequently, $E_\alpha(R) = R_{>0}$ and due to continuity and monotonicity of E_α the inverse function of the restriction function $E_\alpha : R \to R_{>0}$, which is denoted by $\log_\alpha^M : R_{>0} \to R$, exists.

Definition 3 [7] (Fractional Lyapunov Exponent,). Let $f : R_{\geq 0} \to R^d$ be an arbitrary function. The *fractional Lyapunov exponent* of order α of f is defined as [7]:

$$\chi_\alpha(f) = \limsup_{t\to\infty} \frac{1}{t^\alpha} \log_\alpha^M \|f(t)\|.$$

The following theorem provides a practical formulation of $\chi_\alpha(f)$.

Theorem [7]. *Let* $f : R_{\geq 0} \to R^d$ *be an arbitrary function. The following statements hold*:

(*i*) $\chi_\alpha(f) > 0$ *if and only if* $\chi(f) > 0$ *and in this case we get*

$$\chi_\alpha(f) = \chi(f)^\alpha \limsup_{t\to\infty} \left(\frac{1}{t} \log\|f(t)\| \right)^\alpha, \tag{9}$$

where $\chi(f) := \limsup_{t\to\infty} \frac{1}{t} \log\|f(t)\|$.

(*ii*) $\chi_\alpha(f) < 0$ *if and only if* $\limsup_{t\to\infty} t^\alpha \|f(t)\| < \infty$ *and in this case we get*

$$\chi_\alpha(f) = -\frac{1}{\Gamma(1-\alpha)\limsup_{t\to\infty} t^\alpha \|f(t)\|}. \tag{10}$$

(*iii*) $\chi_\alpha(f) = 0$ *if and only if*

$$\chi(f) \le 0 \quad \textit{and} \quad \limsup_{t\to\infty} t^\alpha \|f(t)\| = \infty,$$

where $\chi(f)$ *is the Lyapunov exponent of* f.

Proof. See [9, Theorem 9].

2.3. Entropy and Poincare recurrence

A particularly interesting and deep connection is the one established between metric entropy and Poincare recurrence. Given a measurable dynamical system f, it follows by pioneering work of Poincare that the set of recurrent points has full probability. This means that the iterates of almost every point (with respect to an arbitrary invariant probability measure μ) will return arbitrarily close to itself. In particular, for any positive measure set A the function [11]:

$$R_A(x) = \inf\{k \ge 1 : f^k(x) \in A\} \tag{11}$$

is finite almost everywhere in A/ given a decreasing sequence of partitions U_n it is natural to look for a limiting behavior of the return times R_{U_n} in finer scales. Such a limiting behavior turned out to exist for ergodic stationary processes and it coincides with the metric entropy of the system. Ornstein and Weiss proved that the entropy $h_\mu(f,Q)$ of an ergodic measure μ with respect to a partition Q is given by the (almost everywhere) well defined limit [11]:

$$h_\mu(f,Q) = \lim_{n\to\infty} \frac{1}{n} \log R_n(x,Q), \tag{12}$$

where $R_n(x,Q) = \inf\{k \ge 1 : f^k(x) \in Q^{(n)}(x)\}$ is the *n*th return time (with respect to the partition Q), $Q^{(n)} = \bigcup_{j=0}^{n-1} f^{-j}Q$ is the dynamically generated partition, and $Q^{(n)}(x)$ denotes the element of $Q^{(n)}$ that contains the point x.

2.4. Generalized systems with memory

Let $(\tilde{X}, A^{\alpha}, \Sigma) \subset W$; Z the set of all integers:

$$R_{\geq 0} = [0,\infty),\ R_{\leq 0} = (-\infty,0),\ Z_{\geq 0} = \{1,2,\ldots\}, \text{ and } Z_{\leq 0} = \{0,-1,-2,\ldots\}.$$

Definition 4. Let $(\tilde{X}, A^{\alpha}, \Sigma) \subset W$ be Q. $Q \subseteq R \times Z$ is called a compact generalized memory of

$$Q = Q_{\geq 0} \bigcup Q_{\leq 0}, \tag{13}$$

where
$$Q_{\geq 0} = \bigcup_{j=0}^{j-1} \left([t_j, t_{j+1}], j \right)$$

and
$$Q_{\leq 0} = \bigcup_{k=1}^{k} \left([s_k, s_{k-1}], -k+1 \right)$$

for same finite of observed $s_k \leq \ldots \leq s_i \leq s_0 = 0 = t_0 \leq t_1 \leq \ldots \leq t_j$.

2.5. Poincare recurrence diagram

Displayed the system on the two-dimensional square matrices $[N,N]$ and of formula [12]:

$$R_{i,j}^{m,\varepsilon_i} = \theta\left(\varepsilon_i - \|x_i - x_j\|\right),\ i,j = 1,\ldots,N,\ i \neq j,\ x \in U,$$

where N - number of considered (examined) condition x_i; ε - size of a neighborhood of a point x at the moment i; $\|\cdot\|$ - norm; $\theta(\cdot)$ - function of Heaviside.

2.6. Topological stability of hyperchaotic –order systems

Determine the stability of the zero solution on the system

$$\frac{dx}{dt} = {}_q\hat{\Omega},\ {}_q\hat{\Omega} = \{\omega_n\}_{n=0}^{N},\ {}_q\hat{\Omega} \in_q \hat{\Psi}.$$

Proposition. Let $GM \in U$ be structure of generalized memory. If there exists a differentiable observed $V : R^n \to R^+$ such that the to following hold:

(i) - if trajectory will pass thought the point 0, i.e. $\dot{V}(x) \leq 0$, the system is stable with $d_f(\langle \tau \rangle)$ and matches GM;

(ii) - if trajectory will pass below the point 0, the system is asymptotically stable with $d_f(\langle\tau\rangle)$.

2.7. Topological control

Consider the following general structure of the fractional-order nonlinear system under *control*:

$$ {}_0D_t^q x(t) = f(x(t) + Bu(t)), \quad (14)$$

where $u(t) = [u_1(t)u_2(t)\ldots u_m(t)]^T$ is m- dimensional input vector that will be used and following control structure will be considered for state feedback:

$$u(t) = u_{eq}(t) + u_{sw}(t), \quad (15)$$

where $u_{eq}(t)$ is equivalent control and $u_{sw}(t)$ is the switching control of the system (9).

With regard to the task the topological control will be submitting a number of definitions [13].

Definition 5. The system topologically controllable if and only if coincides with $\hat{x}$ on the basis of the criterion metrics "proximity" Hausdorff.

Theorem 2 [14]. Let E and F is compact subset R^n, $\varepsilon > 0$. Hausdorff distance $H(E,F)$ satisfies the relation.

$$H(E,F) \le \hat{\varepsilon} \Leftrightarrow E \subset F + \hat{\varepsilon} \text{ and } F \subset E + \hat{\varepsilon},$$

where $\hat{\varepsilon} > 0$ the allowable threshold.

Common challenges in the implementation of topological synchronization and topological control is a base – definition, theorem and intelligent iterative algorithm.

Fractional-order iterative learning control scheme is given as [13]:

$$U_{k+p}^{(\alpha)}(t) = F(U_k(t), e_k(t)),$$

where

$$e_k(t) = Y_d(t) - Y_k(t). \quad (16)$$

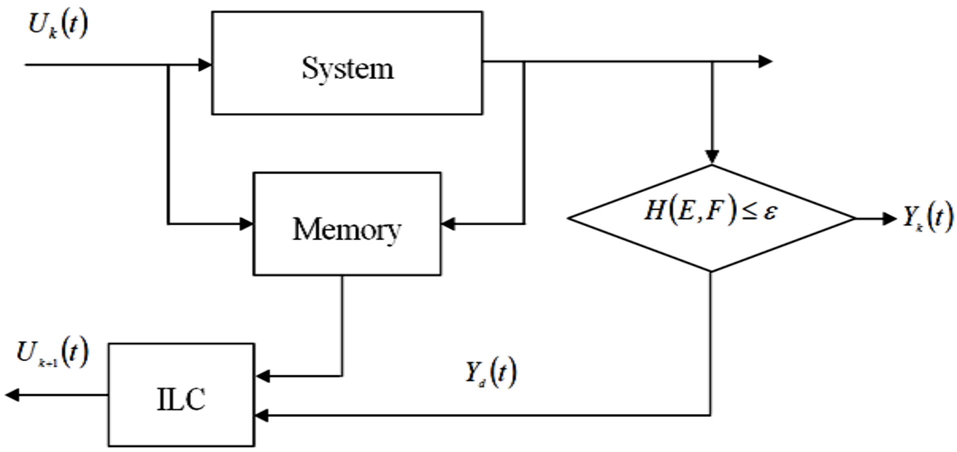


Fig. 1. The basic scheme of iterative learning control with $Y_d(t)$ being the trajectory, $U_k(t)$ and $Y_k(t)$ the input and output signals.

Definition 6. Two systems topologically controllable if and only if they are synchronized topologically.

Remark. If you synchronize at the same time a regularity at the exit system there is a situation called *passive control*.

Otherwise the usual iterative procedure for the organization of regular structure on the system output.

As for Control (*to the control*) it is tracking with the implementation of the T-synchronization so that output of the system occurred Quasiperiodicity. It seems that this problem should be studied in the plane of parameters: "the area of interest – Poincare return time" or "frequency detuning – Poincare return time".

3. Main Results

3.1. Transitive T-synchronization, sliding Control in fractional multidimensional systems with generalize memory

In this section the relation of transitivity, in realization of a problem of synchronization, will be defined as the filter.

Let give three connected fractional hyperchaotic systems.

You want to solve the problem of T-synchronization systems with tracking control.

In general, even if the T-synchronization is implemented under the scheme "transitive relation".

Formal definition. In terms of set theory, the transitive relation can be defined as:

$$\forall a,b,c \in X : (aRb \wedge bRc) \Rightarrow aRc.$$

That is T-synchronized with the first system to second, second to third, then the first and third T-synchronized. T-synchronization is performed by the criterion of "proximity" Poincare return time [8, 13].

Then synchronize transitive relations systems in the context of the Definition and Hausdorff theorem would be:

$$\begin{gathered} \underset{\langle\tau\rangle_{1-2-3}}{Sinc} \overset{prox}{\Leftrightarrow} \left(\langle\tau\rangle_1 \subset \langle\tau\rangle_2 + \varepsilon,\ \langle\tau\rangle_2 \subset \langle\tau\rangle_1 + \varepsilon\right) \\ \wedge\left(\langle\tau\rangle_2 \subset \langle\tau\rangle_3 + \varepsilon,\ \langle\tau\rangle_3 \subset \langle\tau\rangle_2 + \varepsilon\right) \Rightarrow \\ \langle\tau\rangle_1 \subset \langle\tau\rangle_3 + \varepsilon,\ \langle\tau\rangle_3 \subset \langle\tau\rangle_1 + \varepsilon, \end{gathered} \tag{17}$$

where $\langle\tau\rangle_{1-2-3}$ is the average Poincare return times.

T-synchronization process will be considered in the plane of the parameters "area of interest" - proximity capture the average Poincare return time.

In this connection it is of interest the proposal of the qualification effect of the proximity capture the average Poincare return time, i.e.

$$prox\langle\tau\rangle_{GM}^{\alpha} \overset{def}{=} \mu\ , \tag{18}$$

where μ back translation of the coupling coefficient between interconnected systems.

3.2. Fractional-order chaotic filter

Let the interaction the Poincare recurrence $\langle\tau\rangle_{GM}^{\alpha}$ with parameters fractional-order α chaotic is represented as a tupple:

$$\Omega : \langle\tau\rangle_{GM}^{\alpha} \Rightarrow \left(d_f, \lambda, h, D_{KY}, GM\right) \in U\,, \tag{19}$$

where Ω - operator; d_f - fractional dimension; λ - Lyapunov dimension; h - entropy; D_{KY} - Kaplan-Yorke Lyapunov dimension; GM - generalize memory; $U \in \left(X, \langle\tau\rangle\right)$.

Depending of the requirement of the decision-making system filters can be used differentially, i.e. $\langle\tau\rangle\langle\lambda\rangle$, $\langle\tau\rangle\langle h\rangle$ and so on.

Definition 7. Let $U \in (X, \langle\tau\rangle)$ is the compact metric space of Poincare with dimension $\langle\tau\rangle$. Then $F = \langle\tau\rangle_{GM}^{\alpha}$ is a fractional integrated filter with generalized memory if and only if:

1. $\varnothing \in F = \{f_i\}_{i=}^{N}$
2. If $A, B \in F$, then $A \bigcap B \in F$.
3. If $A \in F$, and $A \subseteq B \subseteq u$, the $B \in F$.
4. F satisfies Poincare theorem.

Thus. F is a fractional integrated filter with memory, characterizes the capture of the average Poincare return time.

3.3. Formation of loss memory

It is a known that during the Poincare recurrence characterizes as a "residual", and the real memory of the fractional-order system [15]. Hence the equivalence between the spectrum of the Poincare return time and distribution of generalized memory.

Los memory are determined by the difference between the global and the local fractal dimensions, which means, respectively, reversible and irreversible processes [15]. Loss of information numerically define the entropy.

3.4. Algorithm

Step 1. Let given to the hyperchaotic fractional Liu [16]:

$$\begin{cases} \dfrac{d^{\alpha}x}{dt^{\alpha}} = a(y-x), \\ \dfrac{d^{\alpha}y}{dt^{\alpha}} = -kxz + bx + \vartheta, \\ \dfrac{d^{\alpha}z}{dt^{\alpha}} = dxy - cz, \\ \dfrac{d^{\alpha}\vartheta}{dt^{\alpha}} = -hy, \end{cases} \tag{20}$$

where $a = 10, b = 40, c = 2{,}5,\ d = 4,\ h = 5,\ k = 1$ and $\alpha = 0.9$.

Fractional-order Rabinovich-Fabrikant system following [17]:

$$\left.\begin{aligned} \dot{x}_1 &= x_2\left(x_3 - 1 + x_1^2\right) + \gamma x_1, \\ \dot{x}_2 &= x_1\left(3x_3 + 1 - x_1^2\right) + \gamma x_2, \\ \dot{x}_3 &= -2x_3\left(x_1 x_2 + \alpha\right), \\ \dot{x}_4 &= -3x_3\left(x_2 x_4 + \delta\right) + x_4^2, \end{aligned}\right\} \tag{21}$$

where $\alpha = 0.14,\ \gamma = 1.1,\ -0.01 \le \delta \le 7650$.

The fractional-order Chen system as follows [18]:

$$\begin{aligned} \frac{d^{\alpha}x}{dt^{\alpha}} &= a(y-x) + w, \\ \frac{d^{\alpha}y}{dt^{\alpha}} &= bx - xz + cy, \\ \frac{d^{\alpha}z}{dt^{\alpha}} &= xy - dz, \\ \frac{d^{\alpha}w}{dt^{\alpha}} &= yz + rw, \end{aligned} \tag{22}$$

where $a = 35,\ b = 7,\ c = 12,\ d = 3,\ r = 0.5$ and $\alpha = 0.9$.

Step 2. Let $\hat{x}_{1,2,3_{\alpha}} = \{x_n\}_{n=0}^{N}$ is the mopping of the (1-2) hyperchaotic fractional-order and (3) hyperchaotic systems related transitive relation.

Step 3. On the schema "master-slave" formed by a pair transitive relation

$$(\hat{x}_1 R \hat{x}_2 \wedge \hat{x}_2 R \hat{x}_3) \Rightarrow \hat{x}_1 R \hat{x}_3.$$

Step 4. Produce operation

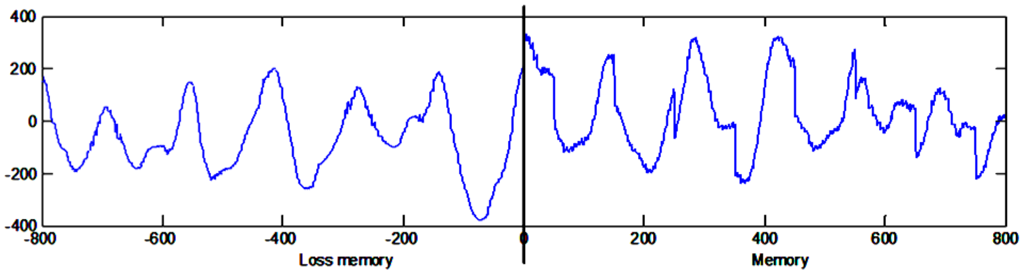
$$(\hat{x}_1 R \hat{x}_2 \wedge \hat{x}_2 R \hat{x}_3) \Rightarrow \hat{x}_1 R \hat{x}_3 \ \vee \ A_{frac}\left(\frac{x}{T}+\varphi\right),$$

where $frac(x)$ is the fractional part, $frac(x) = x - [x]$, A is amplitude, T is the period of the wave, and φ is its phase.

Step 5. Iterative learning algorithm for topological synchronization on schematic "master-slave", and *step 4*, with while tracking control for fractional-order hyperchaotic systems shows in fig.1.

Step 6. On the basis theorem and on iterative procedure define the *effect* of the "proximity" capture of the average Poincare return time as a criterion for the hyperchaotic topologically systems while tracking control.

Step 7. Define the Poincare diagram $D_{\hat{x}_3}$ and average Poincare return time:



Entropy h =7.4358

Fig.2. generalized memory of "master-slave" scheme,

$$\langle\tau\rangle_{Liu} = 4.120, \quad \langle\tau\rangle_{R-F} = 4.133.$$

$$H = \left(\langle\tau\rangle_{Liu} + \varepsilon\right) \subset \langle\tau\rangle_{R-F}, \ \varepsilon = 0.013.$$

Step 8. define Lyapunov stability

(i) Let the Lyapunov function is given in the quadratic form

$$V\left(x_{\substack{R-F \\ mem}}\right) = \frac{1}{2}\Omega^2.$$

(ii) We calculate the total derivative of the function $V(x_{mem})$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} = \hat{x}_{mem}\Omega^2 \le 0 .$$

So the result speaks about asymptotic stability memory function. (fig.3.).

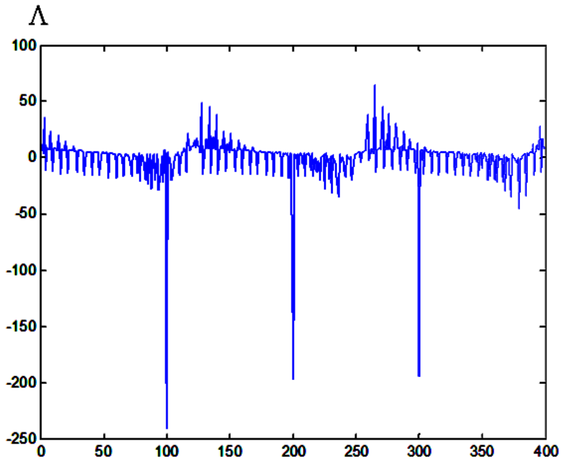


Fig.3. Stability of Lyapunov function.

In the task of relation transitive topological Synchronization hyperchaotic system with tracking control, it was found the transition from hyperchaotic regime to chaotic.

Now it is possible to tell, that the relation transitive topological synchronization hyperchaotic system represents "proximity" of capture of average of Poincare return time, that is the fractional integrated filter

$$F = \langle \tau \rangle_{GM}^{\alpha} = 4.133$$

It should be noted that the proposed non-standard approach in the formation of the filter class, characterized by reversibility property. Numerically filter expresses "proximity" capture the average Poincare return time (fig.4).

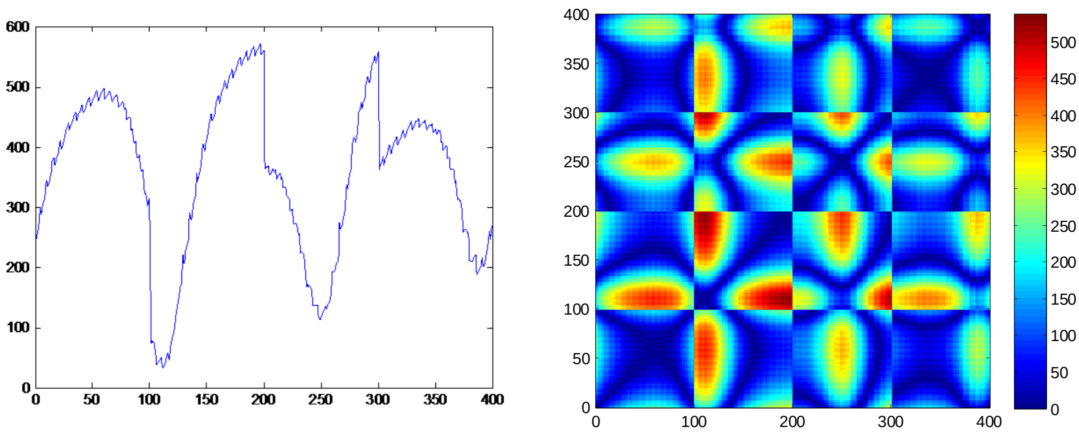


Fig.4 Visualization of fractional filter with memory.

4. Conclusion

In this paper we propose is not a traditional filter, associated with basic characteristics of chaotic dynamical systems.

An example was shown the relation of transitivity, in realization of a problem of synchronization, will be defined as the integrated filter. It demonstrated the transition of hyperchaos - chaos with tracked control and stability. The estimation of the entropy memory loss. The estimate “proximity” of capture of average of Poincare returns time.

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